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PHOTON SERIES

PHYSICS NOTES

1ST YEAR

ACCORDING TO THE NEW SYLLABUS OF KP
TEXT BOOK BOARD (2020 AND ONWARD)
An easy access to the basic concepts of physics

Writers:

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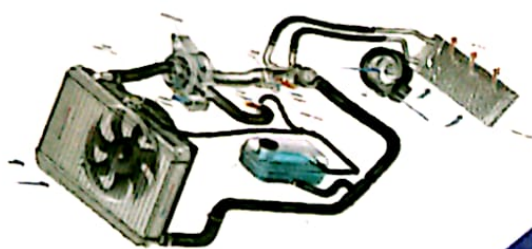
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Salient Features:

- Simple and Easy Text Language
- Separate portions for Exercise and Non-Exercise Long Questions
- Self-explanatory Diagrams
- Solution of MCQ's
- Condense answers of short questions
- Heading Based Short and Long Questions
- Solved Assignments and Numerical Problems





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Table of Contents

Unit No:	Unit Title	Page No
1	Measurement	1
2	Vectors and Equilibrium	34
3	Force and Motion	76
4	Work and Energy	120
5	Rotational and Circular Motion	156
6	Fluid Dynamics	202
7	Oscillation	237
8	Waves	276
9	Physical Optics	334
10	Thermodynamics	371

Exercise Long Questions
Q. 1: Define physics. Explain the scope and importance of physics in science, technology and society.

Ans: **Physics:**

The branch of science which deals with the study of matter, energy and their mutual relationship.

OR

The branch of science which study the behavior of universe.

OR

The branch of science which is concerned with the study of nature.

Scope of Physics:

The scope of physics varies from the smallest subatomic particles (Higgs Bosons) to the largest bodies i.e. stars, galaxies black holes and super cluster. In our daily life we depend on physics for nearly everything. Walking, cooking, cutting, throwing, playing, climbing and constructing a house all involve physics.

If you are sitting in room under the light of a lamp or bulb or enjoying the fresh air of the fan or air conditioner you are encountering physics. It is physics which study and model the laws which govern the motion of electrons and rockets.

Importance of Physics In Science:

Physics is the fundamental scientific discipline which is coordinated or related with chemistry, astronomy, geology and biology. One can say that nearly all natural sciences originate from physics.

Importance of Physics In Technology:

The evolution of the world from Stone Age to the very recent information age is through the advancements in physics. The pivotal discoveries like, laser, television, radio, computer, internet, artificial intelligence, DNA and nuclear weapons are credited to physics.

Importance of Physics In Society:

Physics has revolutionized the life standard of the common man, socially the world has become a global village. Medically the life expectancy has considerably been increased. It is physics which enabled us to live in an age of information.

Q. 2: What is system of units? In SI what is meant by base, derived and supplementary units?

Ans: **SI System or System International:**

It is basically a French word which is a complete, comprehensive and coherent system of units used for the measurement of physical quantities.

Date of Establishment:

It was established in 1960.

History or Purpose:

Due to local system of units used in different regions of the world, there existed a problem in trade and commerce which hindered international trade. It was due to this reason the need for an international system of units was felt.

Other System Before SI System:

Before SI system, other system, MKS (Metre, Kilogram, Second) and FPS (Foot, Pound, Second) were used by scientists.

Base Unit:

The units of base physical quantities are called base units. Base quantities are those quantities which are not derived from other quantities but other quantities are derived from it.

The Number of Base Units:

There are just seven base units.

Tabulation of Base Quantities and Units:

The seven base quantities and their units are tabulated in the following table.

Name	Symbol	Name	Symbol
Length	l, x, r, d	Meter	m
Mass	m	Kilogram	Kg
Time	t	Second	S
Electric current	I	Ampere	A
Temperature	T	Kelvin	K
Amount of substance	n	Mole	mol
Luminous intensity	I _v	Candela	Cd

Derived Units:

The units of those physical quantities which are derived from the base physical quantities are called derived units.

Examples:

- Speed is an example of derived quantity and its unit m/s is an example of derived units.
- The unit of area is square meter (m²) which is an example of derived units.

Supplementary Units:

The two purely geometrical units i.e. radian and steradian were not classified by the scientists either as base or derived units hence there units are called supplementary units. But in 1995, these units were classified as derived units.

Radian:

It is a unit of the plane angle, made by an arc at the center of the circle equal in length to the radius of that circle.

Formula for Finding the Number of Radians:

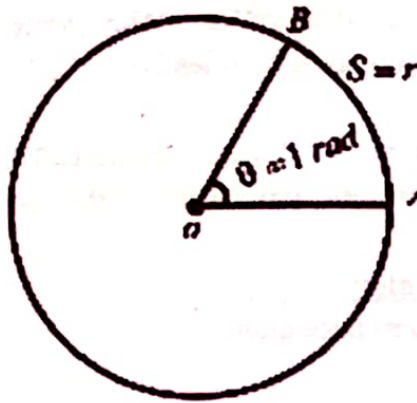
$$\theta = \frac{S}{r}$$

Where θ = Number of radians,

S = Arc length, r = Radius of circle,

Figuratively:

These quantities are figuratively shown in the figure given below.

**Numerically:**

The numerically value of 1 radian is equal to 57.3° .

To Show That $2\pi \text{ rad} = 360^\circ$:

We know that $S = r\theta$ ----- (1)

Also the angle made by circumference at the center is 360° in other words if

$S = 2\pi r \Rightarrow \theta = 360^\circ$ put these values in equation (1).

$$2\pi r \text{ rad} = r 360^\circ$$

OR $2\pi \text{ rad} = 360^\circ$ (r cancels with r) Hence proved.

Steradian:

This is the unit for solid angle. It is the angle made by an area at the center of a sphere, equal to the square of radius of that sphere.

Formula for Finding Number of Steradians:

Number of steradians in a sphere = $\frac{\text{Area of sphere}}{r^2}$

$$= \frac{4\pi r^2}{r^2} = 4\pi$$

**Figuratively:**

Q. 3: What conventions are used in SI to indicate units?

The following conventions are used to indicate units in SI system.

Ans: **Conventions for writing symbols of units:**

- i) Upright (مستقيم) roman letter / alphabets are used as symbol of units regardless of the surrounding text or language.

Example:

m For meter, S, for second, Pa for Pascal.

- ii) Prefix is considered as part of unit and is written without space, with the order that prefix is written first and followed by unit.

Example:

nm, μs etc.

- iii) Compound prefixes or combination of prefixes is not allowed.

Example:

$\mu\mu\text{F}$ should be written as pF.

- iv) The normal rules of mathematics are applicable to the operation of units.
- Multiplication is represented by a space or half-high (centered) dot.
 - Division of units is represented by a horizontal, solidus or oblique or by negative exponents e.g. N.m, N m, m/s or ms^{-1} .
- v) Abbreviation for unit symbols are not allowed.

Example:

It is not permissible to write sec for second, Sq.mm for mm^2 , cc for cm^3 or cubic centimeter.

- vi) The power of unit applies to prefix and unit both.

Example:

$$1m^2 = (100cm)^2$$

Conventions for Writing Unit Names:

- Unit names are treated as ordinary nouns.
- Full name of the name does not begin with a capital letter even if named after a scientist.

Example:

The word "newton" represents unit while Newton represents the name of the scientist Newton.

- The symbol of unit if named after a scientist should be capital and small otherwise.

Example:

Joule is represented by J, hertz by Hz, meter by m, etc.

- The unit name should be spelled out in full.

Example:

2.1 m/s as 2.1 meter/second.

- The names of prefixes and units are written with no space apart.

Example:

Milligram not milli-gram, kilopascal but not kilo-Pascal.

Q. 7: What are errors? Differentiate between systematic and random errors?

Ans: **Error:**

The margin of doubt in a measurement is called error.

OR

The difference of actual value and measured value is called error.

There exists error in each and every measurement.

Types of Errors:

- Systematic Errors:**

The error which tends to be either positive or negative is called systematic error.

Sources of Systematic Errors:

- Instrumental Error:**

These are errors which arise due to imperfect design, poor calibration, or zero error in the apparatus.

b) **Personal Error:**

These errors arise due to personal bias, improper setting of apparatus, carelessness, or neglecting precautions by the experimenter.

Minimization of Systematic Errors:

These errors can be minimized by:

- a) Improving experimental techniques
- b) Use of precise apparatus
- c) Removing personal biasing
- d) Observing precautions

ii) **Random Errors:**

The errors which occur irregularly with respect to size and sign, are called random errors.

Causes of Random Errors:

These errors can arise due to fluctuations in temperature, humidity, wind velocity etc.

Minimization of Random Errors:

These errors can be minimized by taking average of the different readings.

Q. 8: What is uncertainty in measurement? Explain the propagation of uncertainty in addition, subtraction, multiplication and division.

Ans: **Uncertainty:**

The quantification or magnitude of error in a measurement is called uncertainty.

Interpretation: (وضاحت):

Uncertainty is the measure of error in value or measurement. To interpret uncertainty we use the relation:

$$\text{Measurement} = \text{Best Estimate} \pm \text{uncertainty}$$

Let the length of white-board measured by a student is (2.07 ± 0.1) m. the actual value will lie between (2.08) m and (2.06) m.

Type of Uncertainties:

- i) Absolute uncertainty.
- ii) Relative or percent (%) uncertainty.

i) **Absolute Uncertainty:**

The uncertainty in a measurement having the same unit as the measured quantity is called absolute uncertainty.

Symbol:

It is represented by the symbol Δ .

Example:

In the measurement of length of whiteboard which is (2.07 ± 0.1) m. 0.1m is the absolute uncertainty.

ii) **Relative or Percent Uncertainty:**

The ratio of the magnitude of error to the measured value is called relative uncertainty. If the percentage of the relative uncertainty is taken, it is then called percent uncertainty. It has no unit because it is the ratio of two same quantities.

Symbol:

It is represented by the symbol ϵ (epsilon).

Example:

In the measurement of the length of whiteboard which is (2.07 ± 0.1) m the relative or percent uncertainty will be calculated as:

$$\epsilon = \left(\frac{0.1 \text{ m}}{2.07 \text{ m}} \right)$$

$$\epsilon = 0.048$$

While percent uncertainty will be

$$\epsilon = \left(\frac{0.1 \text{ m}}{2.07 \text{ m}} \right) \times 100$$

$$\epsilon = 4.8\%$$

Indicating Uncertainty in Measurement:

Statistical (شماریاتی) methods are often used to calculate uncertainty.

- i) Sum or difference.
- ii) Product or quotient (تقسیم).
- iii) Power
- iv) Finding average
- v) Timing experiment

i) Sum or Difference:

Two values if either added or subtracted the uncertainties will always be added.

Mathematically:

If we are given two quantities A and B and ΔA and ΔB are the uncertainties in these quantities, the according to rule the resultant uncertainty ΔZ in the final value would be calculated as:

For sum and For subtraction:

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

$$Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$$

$$\text{Where } \Delta Z = \pm (\Delta A + \Delta B)$$

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$Z \pm \Delta Z = (A - B) \pm (\Delta A + \Delta B)$$

$$\text{Where } \Delta Z = \pm (\Delta A + \Delta B)$$

ii) Product or Quotient:

If two given quantities are either multiplied or divided the % uncertainties in the quantities will always be added.

If we are given two quantities A and B while $\Delta A\%$ and $\Delta B\%$ are % uncertainties. The uncertainty in the final value or result is calculated as:

Product	Quotient
$Z = A.B$	$z = \frac{A}{B}$
$\Rightarrow Z \pm \Delta Z = (A \pm \Delta A).(B \pm \Delta B)$	$Z \pm \Delta Z = \left(\frac{A \pm \Delta A}{B \pm \Delta B} \right)$
Convert absolute / fractional uncertainties into percent uncertainty.	Convert into percent uncertainties.
$\Rightarrow Z \pm \Delta Z = (A \pm \Delta A\%).(B \pm \Delta B\%)$	$Z \pm \Delta Z = \left(\frac{A \pm \Delta A\%}{B \pm \Delta B\%} \right)$
$\Rightarrow Z \pm \Delta Z = A.B \pm (\Delta A + \Delta B)\%$	$Z \pm \Delta Z = \frac{A}{B} \pm (\Delta A + \Delta B)\%$
Convert back to fractional uncertainty.	Convert into fractional uncertainties.
$\Rightarrow Z \pm \Delta Z = AB \pm \Delta Z$	$Z \pm \Delta Z = \frac{A}{B} \pm \Delta Z$

iii) **Power:**

The percent uncertainty given is quantity is multiplied by power of the quantity to calculate the uncertainty in the measurement.

Mathematically:

If we are given a quantity whose power is n and $\Delta A\%$ is the percent uncertainty in it is:

$$\begin{aligned} \text{If } Z &= A^n \\ \Rightarrow Z \pm \Delta Z &= (A \pm \Delta A\%)^n \\ \Rightarrow Z \pm \Delta Z &= A^n \pm n \times \Delta A\% \\ \Rightarrow Z \pm \Delta Z &= A^n \pm (n\Delta A)\% \end{aligned}$$

Convert again to fractional uncertainty

$$\Rightarrow Z \pm \Delta Z = A^n \pm \Delta Z$$

NOTE:

If the quantities are given but the uncertainties given are fractional, first the uncertainties are converted into percent uncertainties.

Q. 6: What are significant figures? What are the rules for determining the significant figures in the final result after addition, subtraction, multiplication and division?

Ans: **Significant Figures:**

The number of accurately known figures and the first doubtful figure (digits) are known as significant figures.

OR

The digits or numbers which are reliable and taken in consideration in finalizing a measurement or result.

some digits (numbers) are significant while others are not especially when the result is expressed in decimal form. So, only significant figures are valued or taken into account.

General Rules:

1. Non-zero digits will always be significant.

Examples:

The digits from 1 to 9 are always significant. For instance the figure 23.75 has four significant figures.

2. Zeros between two significant digits will be significant.

Examples:

200.03 has five significant figures.

3. Zeros to the left of significant figures are not significant.

Examples:

The number 0.0035 has two significant figures.

4. In decimal fraction (وہ قیمت جس میں اعشاریہ ہو) zeros to the right of a significant figures are significant.

Examples:

5.900 has four significant digits.

5. In a non-decimal fraction, the number of significant figures depends upon the least count of the measuring instrument.

Examples:

In a scale having a least count of 10 kg, the number 20,000 kg has four significant figures.

6. In measurement or result expressed in scientific or standard form the numbers other than the power of 10 will all be significant.

Examples:

The figure 9.11×10^{-31} kg has three significant figures.

Significant Figures in Final Result:

When two or more than two quantities are added, subtracted, divided or multiplied, the final value or result would have as many significant figures as contained by the figure having the least number of significant figures.

Examples:

If we are given three figures / values A, B and C which contain 5, 3 and 2 significant numbers respectively. If these values are calculated by using either addition, subtraction, multiplication or division, the final value must be rounded off up to two significant digits, because the number which has the minimum number of significant figures has two significant digits.

NOTE:

Rounding off of numbers is commonly carried out at the end of calculation.

The smallness of the least count of the device is termed as precision. A meter rod calibrated in centimeters and other in millimeters, the precise measurement would be made by that one calibrated in millimeters.

Accuracy:

The closeness of the measured value to the actual value is called accuracy. Greater the number of significant digits in value higher will be the accuracy of measurement and vice versa.

Difference	
Precision	Accuracy
i. Precision deals with the least count of measuring instrument.	Accuracy is the opposite of uncertainty.
ii. A precise measurement may not be accurate.	An accurate measurement must always be precise.
iii. In a measurement all the digits may or may not be significant.	In an accurate value nearly all digits are significant.

Differentiate With the Help of Darts Diagram:

- If the darts land close to the bull eye and close together, precision and accuracy both are there. Fig (a).
- If the darts land close together but far from bulls-eye there is precision but not accuracy. Fig (b)
- If the darts are spread around the bull eye, there is accuracy but no precision. Fig (c)
- If the darts are spread neither close to each other nor close to bull eye, neither accuracy nor precision is there. Fig (d)



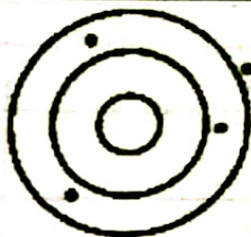
(a)



(b)



(c)



(d)

Q. 8: What are the dimensions of physical quantities? What are limitations and applications of dimensional analysis?

Ans: Dimensions of Physical Quantity:

The nature of a physical quantity are called dimensions of physical a quantity.

OR

An expression of the character of a physical quantity.

Representation of Dimensions:

Dimensions of physical quantities are represented by capital letters or symbols written inside square brackets.

Examples:

The dimensions of length mass and time are represented by [L], [M], [T] respectively.

Dimensions of the Seven Base Quantities:

Quantity	Dimension / Symbol
Length	[L]
Mass	[M]
Time	[T]
Electric current	[I]
Thermodynamic temp	[O]
Amount of substance	[N]
Luminous intensity	[J]

The dimensions of nearly all other quantities are derived from the dimensions of these base physical quantities' dimensions.

Some Important Derived Dimensions:

The dimensions of derived physical quantities are called derived dimensions.

Quantity	Relation	Dimension
Velocity / Speed	$V = \frac{s}{t}$	$\left[\frac{L}{T}\right]$ or $[LT^{-1}]$
Acceleration	$\bar{a} = \frac{\Delta \bar{V}}{\Delta t}$	$[LT^{-1}/T]$ or $[LT^{-2}]$
Force	$\bar{F} = m\bar{a}$	$[M.LT^{-2}]$ or $[MLT^{-2}]$

Quantity	$W = \bar{F} \bar{S}$	$[ML^2T^{-2}]$
Work / Energy	$P = \frac{W}{t}$	$[ML^2T^{-2}/T] \text{ or } [ML^2T^{-1}]$
Power	$A = l \times w$	$[L \times L] \text{ or } [L^2]$
Area	$V = l \times w \times h$	$[L \times L \times L] \text{ or } [L^3]$
Volume	$P = \frac{F}{A}$	$[MLT^{-2}/L^2] \text{ or } [ML^{-1}T^{-2}]$
Pressure	$T = \frac{s}{v}$	$[L/LT^{-1}] \text{ or } [T]$
Time / Period	$F = \frac{1}{T}$	$[\frac{1}{T}] \text{ or } [T^{-1}]$
Frequency		

Limitations of Dimensional Analysis:

- An equation which is dimensionally correct may or may not be physically correct. But a dimensionally incorrect equation must always be physically incorrect.
- The numerical value of the constant of proportionality in other words dimensional constants cannot be found by dimensional analysis.
- The exact relationship among the quantities cannot be found by dimensional analysis.
- Physical quantities having same dimensions cannot be distinguished.
- The vector or scalar nature of a quantity cannot be determined from dimensional analysis.

Applications of Dimensions:

- Dimensional analysis can be used to check the correctness, consistency or homogeneity of an equation.
- Dimensional analysis can be used to determine the power of a quantity.
- Dimensional analysis can be used to determine the relation among physical quantities.

Non-exercise Long Questions

Q. 1: Define scientific notation and highlight its importance.

Ans: Scientific Notation / Standard Form:

It is a mathematical tool to write every big or small number / value in a compact form.

$$\text{Number} = \text{Mantissa} \times 10^{\text{exponent}}$$

OR

$$M = N \times 10^{\pm P}$$

N = Mantissa, P = exponent

Steps for Writing a Number in Scientific Form:

- Only one non-zero digit is left to the right of the decimal point. e.g. 6.23×10^{23} particles, $9.11 \times 10^{-31} \text{ kg}$ etc.
- If the decimal point is moved to the left, the power of 10 increases by the number of places through which the decimal point has been moved.
- If the decimal point is moved to the right, the power of 10 decreases by the number of places through which the decimal point has been moved.

Importance of Scientific Notation:

- It is the quick and shortcut method to write numbers.
- It is an easy and comprehensive way of writing numbers.
- It is an exact and accurate way of writing numbers.

Examples:

The mass of earth in common notation is, 60,000,000,000,000,000,000,000 kg but in scientific notation is given as $6 \times 10^{24} \text{ kg}$ the difference is evident.

Q. 2: Define prefixes to the power of TEN describe its use.

Ans:

Prefixes to the Power of TEN:

Integers to the power of 10 to get a desired unit compatible with the measurement.

Use of prefixes:

Prefixes to the power of 10 convert a non-compatible unit with measurement and measuring device to a compatible and desirable unit.

Prefix	Multiplier	Symbol
Yotta	10^{24}	Y
Zetta	10^{21}	Z
Exa	10^{18}	E
Peta	10^{15}	P
Tera	10^{12}	T
Giga	10^9	G
Mega	10^6	M
Kilo	10^3	K
Hecto	10^2	h
Deca	10^1	da
Deci	10^{-1}	d
Centi	10^{-2}	c
Milli	10^{-3}	m
Micro	10^{-6}	μ
Nano	10^{-9}	n
Pico	10^{-12}	p
Femto	10^{-15}	f
Atto	10^{-18}	a
zepto	10^{-21}	z
Yocto	10^{-24}	y

Q. 3: Define light year, Angstrom and Micron.

Ans: Light Year:

It is the unit of length, and is the length covered by light in vacuum in one year.

Numerically:

One light year is $9.04607 \times 10^{15} \text{ m}$. The diameter of Milky Way galaxy is $2 \times 10^5 \text{ ly}$.

Angstrom ($\text{\AA}$):

The unit of length equal to 10^{-10} meters or 0.1 nanometers. The size of Helium atom is about 1 Angstrom.

Micron:

It is the old name of micrometer equal to 10^{-6} meters. The diameter of RBCs is about 10 microns.

NOTE:

The above three units are non-SI units.

Q. 4: Describe the difference between error and uncertainty.

Ans:

Difference	
Error	Uncertainty
i. It is the difference of actual and measured value.	It is the magnitude of relative error.
ii. Error has the same unit as that of the quantity.	Relative or percent error has no unit.

Q. 5: Define some terms used with dimensions.

Ans: 1. Dimensional Variables:

The physical quantities having dimensions but variable magnitudes are called dimensional variables.

Example: Force, energy, acceleration etc.

2. Dimensional Constants:

The physical quantities having dimensions but constant magnitudes, are called dimensional constant.

Example:

Speed of light in vacuum, planks constant, gravitational constant.

1. Dimensionless Variables:

The physical quantities having no dimensions and variable magnitude are called dimensionless variable.

The physical quantities having no dimensions and constant magnitude are called dimensionless constants.

Example:

Pure numbers, (1, 2, 3.....) the number π etc

Short Questions & Answers

Q. 1: For an answer to be complete, the units need to be specified. Why?

Ans: Reason:

For an answer to be complete, the unit needs to be specified due to the following reasons.

- An answer will be meaningless if units are not specified.
- The answer could mean a variety of quantities or magnitude i.e. m, km, and μm .
- The answer will not give a complete sense and may not be understandable.
- The answer could be interpreted in a variety of ways.

Explanation:

If a person finds the mass of a block to be 5 units of mass. It does not give complete sense because it may be 5kg, 5g or 5 tons. Similarly if the length of a table is measured and found to be 3 units of length. One cannot say whether it is 3 meters, 3 millimeter, or 3 kilometers. The units of meter, millimeter, and centimeter must be specified.

Conclusion:

Thus we find that for an answer to be complete, units must be specified.

Q. 2: What are the advantages of using international system of units?

Ans: The advantages of using the S.I units are given below.

- The S.I units are highly compatible with the measurements.
- These units are easily convertible to higher and lower units by using prefixes (power of ten)
- Almost all the units in S.I are either constructed or defined properly and with high precision e.g. ($1 \text{ mole} = 6.023 \times 10^{23} \text{ atoms / particles}$)
- Samples/copies of these units are at available all over the world.
- This system of units is highly compatible, accepted, accessible and easily understandable even to a common man.
- These units are used by global scientific community as well as by trade and commerce community.

Conclusion:

Thus S.I units are acceptable, available, understandable, compatible and recognized worldwide which gives them an advantage to be used.

Q. 3: How many radians account for circumference of a circle? How many steradians account for surface area of a sphere?

Ans. Radians in a Circle:

The formula to calculate the number of radian is:

$$s = r\theta \quad \text{eq --- (i)}$$

For the circumference of a circle $s = 2\pi r$ put in equation (i).

Equation (i)

$$2\pi r = r\theta$$

$$2\pi(\text{radian}) = \theta$$

or

$$\theta = 2\pi(\text{radian}) \quad A$$

So there are $2\pi \text{ rad}$ in the circumference of a circle.

Steradians in a Sphere:

The formula to calculate the number of steradians is:

$$A = r^2\theta \quad \text{(ii)}$$

The surface area of a solid sphere is ($A = 4\pi r^2$) put in equation (i).

$$4\pi r^2 = r^2\theta$$

$$4\pi(\text{steradians}) = \theta$$

or

$$\theta = 4\pi \text{ sr} \quad b$$

This show that there 4π steradians in the circumference of a solid sphere.

Conclusion:

From equation A and b we conclude that " $2\pi \text{ radians}$ " account for the circumference of a circle and " $4\pi \text{ steradians}$ " account for the circumference of a sphere.

Q. 4: What is least count error? How can least count error be reduced?

Least Count Error:

- It is defined as an instrumental (systematic) error related to the precision.
- This error arises due to the least count of a measuring instrument. An instrument cannot measure a value less than its least count.
- This error may also arise from the resolution of an instrument.

Explanation:

The smallest possible value measured by an instrument is equal to the least count. Measured values are acceptable only up to that value. Also, the approximation made by an observer when the object falls between the smallest divisions of the scale lead to this type of error.

Example:

While measuring the length of a table if one end of the table coincides with zero and the other falls within 2.5 and 2.6 cm. then an approximation is made about the length of table say 2.55 cm.

Reducing Least Count Error:

A least count error can be reduced by the following methods.

- By using an instrument of high precision. A precise instrument is one that has smaller least count.
- Repeating the observation many times and taking the arithmetic mean (Average).

Conclusion:

The error arising from the least count of an instrument is called a "Least count error". And this error can be reduced by:

- Using highly precise instrument and by repeating the measurement.

Q. 5: Why including more digits in an answer does not make it more accurate?

Ans: **Reason:**

Including more digits in an answer does not make it more accurate due to the following reasons.

- Number of digits in answer only increases the precision and not the accuracy.
- Accuracy has nothing to do with the number of digits, it only shows how close an answer is to the actual value.
- More number of digits in an answer means that the measurement has been carried out with a highly precise instrument (Instrument having smaller least count).
- More digits means the measurement is more precise but a precise measurement does not imply that the measurement is accurate as well.

Conclusion:

Including more digits in an answer does not make it more accurate because accuracy depends on the other techniques of measurement and not on the number of digits

Q. 6: What determines the precision of a measurement?

Ans: **Determination of Precision:**

The precision of a measurement is determined by:

- The number of digits in a measurement. More the number of digits more precise the measurement will be.
- Precision can also be determined by the least count of the instrument.

Explanation:

More number of digits in a measurement means it is more precise. But the number of digits depends on the least count of the instrument used.

If the least count of the instrument is small there will be more digits in the measurement and as a result it will be more precise.

Example:

The least count of screw gauge is (0.01mm) so it will measure length upto 2 (two) decimal places. On the other hand the least count of Vernier caliper is (0.1mm). So it can measure a length upto only one decimal place.

Thus the number of digits in answer found by a screw gauge is more than the number of digits in the answer found by the Vernier caliper.

Q. 7: If two quantities have different dimensions, Is it possible to multiply and / or divide, Can we add and / or subtract them?

Ans: Division and Multiplication:

- Yes, if two quantities have different dimensions it is possible to multiply and divide them.
- If A and B are two physical quantities with different dimensions then
 $A \cdot B = \text{Correct operation}$
 $A/B = \text{Correct operation}$

Reason:

Multiplication and division are possible because, multiplication and division of such quantities result in other physical quantities.

Example:

$$\vec{r} \times \vec{F} = \vec{\tau} \quad \left[\vec{r} = \text{moment arm } [L] \text{ and } \vec{F} = \text{force } \left[\frac{ML}{T^2} \right] \right]$$

$$\text{And } \frac{\vec{S}}{t} = \vec{V} \quad \left[\vec{S} = \text{displacement } [L] \text{ \& } t = \text{time } [T] \right]$$

Division result another physical quantity called velocity $[LT^{-1}]$

Addition and Subtraction:

- The addition and subtraction of two quantities with different dimensions is not possible.
- If A and B are two quantities then,
 $A + B = \text{Incorrect operation}$
 $\& A - B = \text{Incorrect operation}$

Reason:

Addition and subtraction of two different quantities is not possible because:

- Addition and subtraction of two different quantities does not result in other physical quantities.

Example:

$$\vec{r} + \vec{F} = \text{undefined}$$

$$\vec{S} - t = \text{undefined}$$

Conclusion:

Multiplication and division of two quantities with different dimensions is possible but addition and subtraction is not possible.

Q. 8: The human pulse and the swing of a pendulum are possible time units. Why are they not often used?

Ans: Reason:

The human pulse and the swing of pendulum are not often used as time units due to the following limitations.

Limitations of Human Pulse:

- The pulse rate of humans vary from person to person.
- It varies with age. Therefore the pulse rate of a child is faster than an adult person.
- It also changes in different conditions of the human body. Like running, sleeping.
- It also varies with the condition of mental health like:
 - Anger (غضب)
 - Happiness
 - Fear
 - Sadness

Conclusion (1):

Due to the above mentioned limitations human pulse is not often used as time unit.

Limitations of Pendulum:

These are given below:

- The time period of pendulum is given by $T = 2\pi \sqrt{\frac{l}{g}}$
- Time period depends on the length of the string and the value of "g".
- The length "l" may change with temperature.
- The value of "g" varies with altitude.
- Air resistance is always present to stop the pendulum.

Conclusion (2):

The time period of a pendulum will change due to the above mentioned reasons and will not remain the same. Therefore it is not often used as unit of time.

Q. 9: If an equation is dimensionally correct, Is that equation a right equation?

Ans: If an equation is dimensionally correct, it does not necessarily mean that the equation is a right equation.

Reason:

- A dimensionally correct equation may or may not be a physically right equation.
- It is one of the limitation of dimensional analysis that it does not tell anything about the scalar and vector nature of a quantity.
- A physically right equation must be dimensionally correct but the converse is not true.

Example:

Work and torque have same dimensions so the equation.

equations are:

- Momentum and impulse.
- Angular frequency & angular velocity.

Conclusion: A dimensionally correct equation may or may not be a right equation.

MCQ's

1. What is the radian measure between the arms of watch at 5:00 pm?
A. 1 radian B. 2 radian C. 3 radian D. 4 radian
2. $1^\circ =$ _____
A. 0.01745 radian B. 1 radian C. 3.14 radian D. 2π radian
3. The metric prefix for 0.000001 is
A. hecto B. micro C. deca D. nano
4. Which of the following is the correct way of writing units?
A. 71 Newton B. 12 μs C. 8 Kg D. 43 kg m^{-3}
5. A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as
A. $(49.8 \pm 0.2) \text{ cm}$. B. $(49.8 \pm 0.4) \text{ cm}$.
C. $(50.0 \pm 0.2) \text{ cm}$. D. $(50.0 \pm 0.4) \text{ cm}$.
6. The percent uncertainty in the measurement of $(3.76 \pm 0.25) \text{ m}$ is
A. 4% B. 6.6% C. 25% D. 376%
7. The temperatures of two bodies measured by a thermometer are $t_1 = (20 \pm 0.5)^\circ\text{C}$ and $t_2 = (50 \pm 0.5)^\circ\text{C}$. The temperature difference and the error therein is
A. $(30 \pm 0.0)^\circ\text{C}$ B. $(30 \pm 0.5)^\circ\text{C}$ C. $(30 \pm 1)^\circ\text{C}$ D. $(30 \pm 1.5)^\circ\text{C}$
8. $(5.0 \text{ m} \pm 4.0\%) \times (3.0 \text{ s} \pm 3.3\%) =$
A. $15.0 \text{ ms} \pm 13.2\%$ B. $15.0 \text{ ms} \pm 7.3\%$
C. $15.0 \text{ ms} \pm 0.7\%$ D. $15.0 \text{ ms} \pm 15.3\%$
9. $(2.0 \text{ m} \pm 2.0\%)^3 =$
A. $8.0 \text{ m}^3 \pm 1.0\%$ B. $8.0 \text{ m}^3 \pm 2.0\%$
C. $8.0 \text{ m}^3 \pm 5.0\%$ D. $8.0 \text{ m}^3 \pm 6.0\%$
10. The number of significant figures in measurement of 0.00708600 cm are
A. 3 B. 4 C. 6 D. 9
11. How many significant figures does $1.362 + 25.2$ have?
A. 2 B. 3 C. 5 D. 8
12. Compute the result to correct number of significant digits $1.513 \text{ m} + 27.3 \text{ m} =$
A. 29 m B. 28.8 m C. 28.82 m D. 28.813 m
13. If 7.365 and 4.81 are two significant numbers. Their multiplication in significant digits is:
A. 36.72435 B. 36.724 C. 36.72 D. 36.7

14. The precision of the measurement 385,000 km is
 A. 10 km B. 100 km C. 1000 km D. 1000000 km
15. $[M^1 L^1 T^1]$ are dimension of
 A. strain B. refractive index C. magnification D. All of these
16. The dimensions of torque are
 A. $[MLT]$ B. $[M^2 L^2 T]$ C. $[ML^2 T^2]$ D. $[ML^2 T^2]$

Solution of MCQs

1. The sum of all angles in clock (circle) is 360° .

It means

$$12 \text{ hrs} = 360^\circ$$

$$1 \text{ hr} = 30^\circ$$

The angle between 5 and 12 is $30^\circ (5) = 150^\circ$

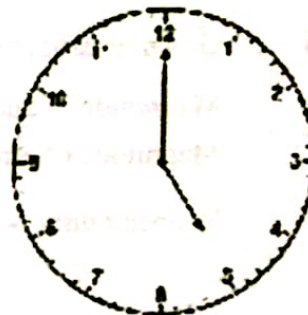
One radian = 57.3°

So the number of radians between 5 and 12 or for 150° are given as.

$$\text{No of radians} = \frac{150^\circ}{57.3^\circ}$$

$$\text{No of radians} = 2.6 \approx 3 \text{ radians}$$

Option C is correct.



2. $1^\circ = ?$

We know that

$$2\pi \text{ rad} = 360^\circ$$

or

$$360^\circ = 2\pi \text{ rad}$$

$$1^\circ = \frac{2\pi}{360^\circ}$$

$$1^\circ = \left(\frac{2 \times 3.14}{360^\circ} \right)$$

$$1^\circ = 0.0174^\circ$$

Option A is correct.

3. 0.000001 in scientific notation is 1.0×10^{-6} and is called micro and its symbol is μ .

Option B is correct.

4. Option D is correct i.e. 43 kg m^{-3}

5. Since the student has measured the distance several times so the error is random or systematic error. And to minimize this error we will take average value and the average of 49.9cm and 50.2cm.

$$\text{average} = \frac{49.8 \pm 50.2}{2}$$

$$\text{average} = 50 \text{ cm}$$

So it is

$$(50 \pm 0.2) \text{ cm}$$

Option "C" is correct.

6. Given measurement is $(3.76 \pm 0.25) \text{ m}$

$$\text{Magnitude of quantity} = 3.76 \text{ m}$$

$$\text{Magnitude of error} = \pm 0.25 \text{ m}$$

$$\% \text{ uncertainty} = \frac{\text{magnitude of error}}{\text{magnitude of quantity}}$$

$$\% \text{ uncertainty} = \frac{100}{100} \times \frac{0.25}{3.76}$$

$$\% \text{ uncertainty} = 6.6\%$$

Option "B" is correct

7. Given temperatures

$$t_1 = (20 \pm 0.5)^\circ\text{C} \quad t_2 = (50 \pm 0.5)^\circ\text{C}$$

Where

$$t_1 = 20^\circ\text{C} \quad \Delta t_1 = \pm 0.5^\circ\text{C}, \quad t_2 = 50^\circ\text{C}, \quad \Delta t_2 = \pm 0.5^\circ\text{C}$$

$$\text{Difference} = t_2 - t_1$$

$$\text{Difference} = 50 - 20$$

$$\text{Difference} = (30^\circ\text{C} \pm 1)^\circ\text{C} \quad \text{option "C" is correct.}$$

Because we know that the uncertainties added in case of difference or subtraction.

8. Given that: $(5.0 \text{ m} \pm 4.0\%) \times (3.0 \text{ s} \pm 3.3\%) = ?$

$$\Rightarrow 5.0 \text{ m} \times 3.0 \text{ s}$$

$$\Rightarrow 15 \text{ ms} \pm 7.3\%$$

Because % uncertainties would be added in case of multiplication.

Option "B" is correct.

$$\frac{4}{100} \times 5 = 0.2$$

$$\frac{3.3}{3} = 0.033$$

+

$$15 \pm 0.233$$

9. Given that: $(2.0 \text{ m} \pm 2.0\%)^3 = ?$

In case of power of a quantity, uncertainty is found by multiplying power with percent. Uncertainty. So uncertainty would be:

$$\begin{aligned}\text{Uncertainty} &= \text{power} \times 2.0\% \\ &= 3 \times 2\% = 6\% \\ &\Rightarrow 8.0m^3 \pm 6.0\%\end{aligned}$$

Hence option D is correct.

10. **Given that:** 0.0070860

The no of significant figures is six or (6)

Option "C" is correct.

11. **Given that:** $1.362 + 25.2 = ?$

$$\begin{array}{r} 1.362 \\ + 25.2 \\ \hline 26.562 \end{array}$$

According to rule for significant figures. The final result would contain as many significant numbers as contained by the number having the least number of significant figures.

As $1.362 = 4$, significant figures

$25.2 = 3$, significant figures

So answer will here three significant figures

$26.6 = 3$, significant figures

Option "B" is correct.

12. **Given that:** $1.513m + 27.3m =$

$$\begin{array}{r} 1.513m \\ + 27.3m \\ \hline 28.813m \end{array}$$

$1.513 = 4$, significant figures

$27.3 = 3$, significant figures

The answer will contain as many significant figures as contained by the number having the least number of significant figures.

So according to rule the answer would contain three significant figures i.e. 28.8m.

Option "B" is correct.

13. **Given that:** $7.635 = 4$, significant figures

$2.81 = 3$, significant figures

$$7.635 \times 4.81 = 36.72435$$

of some quantities) so $M^0 L^0 T^0$ are the dimensions of all the mentioned quantities. All of these.

Option "D" is correct.

16. Torque = $\vec{r} \times \vec{F}$

Or Torque = $\vec{r} \times m\vec{a}$

Torque = $\vec{r} \times m \frac{V}{t}$

Put dimensions

$$[Torque] = [L] \left[M L \frac{T^{-1}}{T} \right]$$

$$[Torque] = [ML^2 T^{-2}]$$

Option "C"

Assignments Solution

Assignment 1.1: A pulley has a radius 0.9m is used to lift a bucket from the well. If it took 3.6 rotations for the pulley to take water out of the well, how deep is water in the well?

Given Data

Number of rotation = $n = 3.6$

Radius = $r = 0.9 \text{ m}$

Radian in one rotation = $\theta = 2\pi \text{ rad}$

Radian in 3.6 rotation = $n\theta = (3.6) 2\pi \text{ rad}$

Required Data

Depth = $S = ?$

Solution:

As we know that

$$S = r\theta$$

For n numbers of rotation it becomes

$$S = rn\theta \text{ ----- (*)}$$

Putting values in eq (*) we get

$$S = (0.9\text{m})(3.6 \times 2\pi \text{ rad})$$

Result:

$$S = 20\text{m}$$

Assignment 1.2 A physicist calculated the wall width of half brick thickness (the brick is laid in flat position, lengthwise called stretcher position), as $(13.6 \pm 0.1) \text{ cm}$. And one brick thickness (the brick is placed in flat position, lengthwise orthogonal to wall, called header position), as $(23.6 \pm 0.1) \text{ cm}$. Calculate the difference in width of walls with uncertainty in it.

Given Data:

Width of the brick in stretcher position = $d_1 = (13.6 \pm 0.1) \text{ cm}$

Width of the brick in header position = $d_2 = (23.6 \pm 0.1) \text{ cm}$

Required Data:

Difference in width (with uncertainty) = $d = ?$

Solution: To find "d" we would subtract " d_2 " from " d_1 " however we will add its fractional uncertainties as

$$d = (d_2 - d_1) \pm (\Delta d_1 + \Delta d_2)$$

Putting values

$$d = (23.6 - 13.6) \text{ cm} \pm (0.1 + 0.1) \text{ cm}$$

Result: $d = (10.0 \pm 0.2) \text{ cm}$

Assignment 1.3 The voltage ' $V (V + \Delta V)$ ' is measured as $(7.3 \text{ v} \pm 0.1 \text{ v})$ and current ' $I (I + \Delta I)$ ' is measured as $(2.73 \text{ A} \pm 0.05 \text{ A})$. Calculate the resistance 'R' by using

Ohm's law as, $R = \frac{V}{I}$.

Given Data:

Voltage = ' $V (V + \Delta V)$ ' = $7.3 \text{ v} \pm 0.1 \text{ v}$

Current = ' $I (I + \Delta I)$ ' = $2.73 \text{ A} \pm 0.05 \text{ A}$

Required Data:

Resistance = $R = ?$

Solution:

$$\% \text{ uncertainty in } V = 7.3 \pm \frac{0.1}{7.3} \times 100\%$$

$$= 7.3 \pm 1.37\%$$

$$\% \text{ uncertainty in } I = 2.73 \pm \frac{0.05}{2.73} \times 100\%$$

$$= 2.73 \pm 1.83\%$$

Since

$$V = IR$$

$$R = \frac{V}{I}$$

Putting values

$$R = \frac{7.3}{2.73} \pm (1.37\% + 1.83\%)$$

Converting percentage uncertainty into fractional uncertainty

$$R = (2.7 \pm 3.2\%) \Omega = 2.7 \pm \frac{3.2}{100} \times 2.7 = (2.7 \pm 0.08) \Omega$$

Result:

$$R = (2.7 \pm 0.08) \Omega$$

Assignment 1.4 The radius of sphere 'r' is measured with vernier calipers as $(r \pm \Delta r) = (2.25 \pm 0.01) \text{ cm}$. calculate the volume of sphere.

Given Data:

$$\text{Radius} = r = (2.25 \pm 0.01) \text{ cm}$$

$$\begin{aligned} \% \text{ uncertainty in } r &= 2.25 \pm \frac{0.01}{2.25} \times 100\% \\ &= 2.25 \pm 0.4\% \end{aligned}$$

Required Data:

$$\text{Volume of sphere} = V = ?$$

Solution:

$$\text{Since } V = \frac{4}{3} \pi r^3$$

Putting values

$$V = \frac{4}{3} \pi (2.25) \pm 3(0.4\%)$$

$$V = \frac{4}{3} (3.14) (2.25) \pm 1.3\%$$

$$= 47.7 \pm 1.3\%$$

$$= 47.7 \pm \frac{1.3}{100} \times 47.7$$

$$= (47.7 \pm 0.6) \text{ cm}^3$$

NOTE

In case of power of a quantity uncertainty is found by multiplying power with percent uncertainty.

Converting %

Uncertainty into

Fractional uncertainty

Assignment 1.5:

Calculate the answers to the appropriate number of significant figures.

- (a) $0.31 + 0.1$
- (b) $658.0 + 23.5478 + 1345.29$
- (c) 8×7
- (d) $0.9935 \times 10.48 \times 13.4$
- (e) $5.5/1.1$
- (f) $\frac{73.2 + 18.72 \times 6.1}{3.4}$

(a) 0.4, (b) 2026.8, (c) 60 (d) 140 or 1.40×10^2 , (e) 5.0 & (f) 55

Solution:

a) $0.31 + 0.1$

Sol: 0.41

Since the number with least significant digits is 0.1.

So the answer will be 0.4.

b) $658.0 + 23.5478 + 1345.29$

Sol: = 2,026.8378

Since the number with least significant digits is (658.0)

So, the answer will be rounded off to, four digits.

= 2026.8

- c) 8×7
 $= 56$
 $= 60$
- d) $0.9935 \times 10.48 \times 13.4$
 $= 139.519192$
 $= 140$
- e) $5.5/1.1$
 $= 5.0$
- f) $\frac{73.2 + 18.72 \times 6.1}{3.4}$
 $= 55.1152$
 $= 55$

Assignment 1.6:

Show that the equations (a) $v_f = v_i + at$ (b) $s = v_i t + \frac{1}{2} at^2$ are dimensionally correct.

a) $V_f = V_i + at$

To show that it is dimensionally correct for this take

$$L.H.S = [V_f] = [m/s] = [ML^{-1}T^0] \text{-----} (*_1)$$

$$\begin{aligned} R.H.S = [V_i + at] &= \left[m/s + \frac{m}{\cancel{\text{sec}^2}} \cdot \cancel{\text{sec}} \right] \\ &= [m/s + m/s] \\ &= [2m/s] \quad (2 \text{ is dimensionless}) \\ &= [ML^{-1}T^0] \text{-----} (*_2) \end{aligned}$$

From eq $(*_1)$ & $(*_2)$ it is clear that

$$\dim [] R.H.S = \dim [] L.H.S$$

Hence $V_f = V_i + at$ is a dimensionally correct relation

b) $S = V_i t + \frac{1}{2} at^2$

$$L.H.S = [S] = [m] = [ML^0T^0] \text{-----} (*_3)$$

$$\begin{aligned} R.H.S = \left[V_i t + \frac{1}{2} at^2 \right] &= \left[\frac{m}{\cancel{s}} \cdot \cancel{s} + \frac{1}{2} \frac{m}{\cancel{s^2}} \cdot \cancel{s^2} \right] \\ &= \left[m + \frac{1}{2} m \right] = \left[\frac{3}{2} m \right] \\ &= [m] = [ML^0T^0] \text{-----} (*_4) \end{aligned}$$

From eq $(*_3)$ & $(*_4)$ it is clear that

$$\dim [] L.H.S = \dim [] R.H.S$$

$\left(\frac{3}{2} \right)$ is a dimensionless constant

Hence $S = Vit + \frac{1}{2}at^2$ is a dimensionally correct relation

Assignment 1.7:

Find an expression for the time period 'T' of a simple pendulum. The time period 'T' may depend upon (i) mass 'm' of the bob of the pendulum, (ii) length 'l' of pendulum, (iii) acceleration due to gravity 'g' at the place where the pendulum is suspended.

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Given Data:

Time period "T" of simple pendulum depends upon

- Mass "m" of the bob
- Length "l" of the pendulum
- Acceleration due to gravity "g"

Required Data:

Possible formula for time period "T" = ?

Solution: Let $T \propto m^a l^b g^c$ ----- (*)

$$T = km^a l^b g^c \text{ ----- (*)}$$

Where K is constant of proportionality and is equal to $K = 2\pi$

Putting appropriate dimensions in eq (*)

$$[M^0 L^0 T] = [M]^a [L]^b [L T^{-2}]^c$$

$$[M]^0 [L]^0 [T]^1 = [M]^a [L]^b [L]^c [T]^{-2c}$$

Comparing the powers of similar physical quantities.

$$a = 0$$

$$b + c = 0$$

$$-2c = 1 \Rightarrow c = -\frac{1}{2}$$

$$b + c = 0 \Rightarrow b - \frac{1}{2} = 0 \Rightarrow b = \frac{1}{2}$$

Putting values of a, b & c in eq (*) we get

$$T = Km^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

$$T = 2\pi (1) \left(\frac{l}{g} \right)^{\frac{1}{2}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{l}{g}}$$

Result:

Numerical Problems

Numerical problem 1: A circular pizza into 3 equal parts, one piece of pizza is taken out. Estimate the degree measure of the single piece of pizza and convert the measure into radians. What is the radian measure of the angle of the remaining part.

Given Data:

Circular pizza cut into 3 equal parts,
One piece of pizza is taken out

Required Data:

- Degree in one piece =?
Also convert degree into radian
- Radian measure of the angle of remaining part =?

Solution:

a) As, total degrees in pizza (circle) = 360°

$$\text{Degrees in one piece} = \frac{360}{3} = 120^\circ$$

To convert degree into rad

$$\text{As } 2\pi \text{ rad} = 360^\circ$$

$$\frac{2\pi \text{ rad}}{360} = \frac{360^\circ}{360}$$

$$1^\circ = \frac{2\pi \text{ rad}}{360} \quad (\pi = 3.14)$$

$$1^\circ = 0.017 \text{ rad}$$

$$120 \times 1^\circ = 120 \times 0.017 \text{ rad}$$

$$120^\circ = 2.09 \text{ rad}$$

This is the radian measurement of one piece.

b) The radian measure of remaining

$$\text{Two pieces will} = 2 \times (\text{radian measure of 1 piece})$$

$$= 2 \times (2.09) \text{ rad}$$

$$\boxed{= 4.18 \text{ rad}}$$

Numerical problem 2: The length of a pendulum is $(1.5 \pm 0.01) \text{ m}$ and the acceleration due to gravity is taken into account as $(9.8 \pm 0.1) \text{ m s}^{-2}$. Calculate the time period of the pendulum with uncertainty in it.

$$(2.5 \pm 0.8\%)$$

Given Data:

$$\text{Length of pendulum} = l = (1.5 \pm 0.01) \text{ m}$$

$$\text{Acceleration due to gravity} = g = 9.8 \text{ m / sec}^2$$

Required Data:

Time period = (with uncertainty) =?

Solution:

$$\% \text{ uncertainty in } l = 1.5 \pm \frac{0.01}{1.5} \times 100\%$$

$$= 1.5 \pm 0.66\%$$

$$\% \text{ uncertainty in } g = 9.8 \pm \frac{0.01}{9.8} \times 100\%$$

$$= 9.8 \pm 0.102\%$$

Since

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2(3.14) \sqrt{\frac{1.5}{9.8}} \pm (0.66\% + 0.102\%)$$

$$T = 6.28(0.387) \pm 0.76\%$$

$$T = 2.4 \pm 0.8\%$$

$$T = 2.4s \pm \frac{0.8}{100} \times 2.4s$$

$$T = 2.4s \pm 0.02s$$

Numerical problem 3: Determine the area of a rectangular sheet with length with length
 $(l \pm \Delta l) = (1.50 \pm 0.02)m$ and width $(W \pm \Delta W) = (0.20 \pm 0.01)m$ Calculate the area $(A \pm \Delta A)$.
Given Data:

$$l \pm \Delta l = (1.50 \pm 0.02)m$$

$$W \pm \Delta W = (0.20 \pm 0.01)m$$

Required Data:

$$(A \pm \Delta A) = ?$$

Solution:

$$\% \text{ uncertainty in } l = \frac{\Delta l}{l} \times 100 = \frac{0.02}{1.50} \times 100$$

$$= 1.33\%$$

$$l \pm \Delta l = 1.50 \pm 1.33\%$$

$$\% \text{ uncertainty in } W \pm \Delta W = \frac{\Delta W}{W} \times 100 = \frac{0.01}{0.20} \times 100 = 5\%$$

$$W \pm \Delta W = 0.20 \pm 5\%$$

As area of rectangle = $l \times w$

$$A = l \times w$$

$$A = (1.50 \pm 1.33\%)(0.20 \pm 5\%)$$

$$A = (1.50 \times 0.20) \pm (1.33\% + 5\%)$$

In multiplication % uncertainties are added

$$A = (0.30 \pm 6.33\%)m^2$$

Convert back to fractional uncertainty

$$A = 0.30 \pm \frac{6.33}{100} \times 0.30$$

$$A = 0.30 \pm 0.01899 = (0.30 \pm 0.02) m^2$$

Numerical problem 4: Calculate the answer up to appropriate numbers of significant digits

(a) $246.24 + 238.278 + 98.3$

(b) $1.4 \times 2.639 + 117.25$

(c) $(2.66 \times 10^4) - (1.03 \times 10^3)$

(d) $(112 \times 0.456) / (3.2 \times 120)$

(e) 168.99×9

(f) $1023 + 8.5489$

(a) 582.8, (b) 120.9, (c) 2.56×10^4 , (d) 0.13, (e) 2000 & (f) 1032

Solution:

a) $246.24 + 238.278 + 98.3$

$$246.24$$

$$238.278$$

$$98.3$$

$$\boxed{582.818}$$

Since the number with least significant digits is 98.3,

So the answer will be 582.8

b) $1.4 \times 2.639 + 117.25$

$$1.4 \times 2.639 + 117.25$$

$$= 3.6946 + 117.25$$

$$= 120.9446 = 120.9$$

The number initially with least significant digit is 1.4

So the answer will be 120.9

c) $(2.66 \times 10^4) - (1.03 \times 10^3)$

$$2.66 \times 10^4 - 1.03 \times 10^3$$

$$= 26600 - 1030$$

$$= 25570$$

$$= 2.5570 \times 10^4 = 2.56 \times 10^4$$

In both given numbers the significant figures are three

d) $(112 \times 0.156) / (3.2 \times 120)$

$$= 51.072 / 384$$

$$= 0.13$$

Since the number with the least number of significant digit is (3.2) having two significant digit so, answer will become = 0.13

e) 168.99×9

$$= 1520.91$$

Since there is only one significant figure in the 2nd value so, the answer must be rounded off to only one significant digit.

$$= 1520.91 = 1521 = 1521$$

$$= 2000 \text{ (Rounded off)}$$

f) $1023 + 8.5489$
 $= 1031.5489$

The number of significant digits in the least precise value is four. So, the answer must be rounded off to four significant digits.

$$\text{Result} = 1032$$

Numerical problem 5: Calculate the answer up to appropriate numbers of significant digits (a) The ratio of mass of proton ' m_p ' to the mass of electron ' m_e '

$$\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1096 \times 10^{-31} \text{ kg}}$$

(b) The ratio of charge on electron ' q_e ' to mass of electron ' m_e '

$$\frac{q_e}{m_e} = \frac{1.6 \times 10^{-19} \text{ C}}{9.1096 \times 10^{-31} \text{ kg}}$$

Solution:

a) $\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1096 \times 10^{-31} \text{ kg}}$
 $= 0.18332 \times 10^4$
 $= 1.8332 \times 10^3$

Answer needs to be rounded off to three digits.

Result a) $= 1.83 \times 10^3$

b) $\frac{q_e}{m_e} = \frac{1.6 \times 10^{-19} \text{ C}}{9.1096 \times 10^{-31} \text{ kg}}$
 $= 0.17563 \times 10^{12} \text{ C/kg}$
 $= 1.7563 \times 10^{11} \text{ C/kg}$

Rounding off to two digits.

Result b) $= 1.8 \times 10^{11} \text{ C/kg}$

Numerical problem 6: Find the dimensions of

(a) Planck's constant ' h ' from formula $E = hf$
 Where E is the energy and f is frequency.

(b) gravitational constant ' G ' from the formula $F = G \frac{m_1 m_2}{r^2}$

$$E = hf$$

$$\Rightarrow E = f \left(\frac{1}{T} \right) \quad \left(f = \frac{1}{T} \right)$$

$$\Rightarrow h = ET$$

$$[h] = [ET] \quad E = \text{Energy or work}$$

T = Time

$$[h] = [E][T]$$

$$[h] = [W][T]$$

$$= [F.S][T]$$

$$= [m.a.S.T]$$

$$= \left[\frac{\text{kg} \cdot \text{m}}{\text{sec}^2} \cdot \text{m} \cdot \text{Sec} \right]$$

$$= \left[\frac{\text{kg} \cdot \text{m}^2}{\text{Sec}} \right]$$

$$[h] = [ML^2T^{-1}]$$

b) Given that:

$$F = G \frac{m_1 m_2}{r^2}$$

Required:

$$[G] = ?$$

$$F = G \frac{m_1 m_2}{r^2}$$

$$G = \frac{Fr^2}{m_1 m_2}$$

$$[G] = \left[\frac{m a r^2}{m_1 m_2} \right]$$

$$[G] = \left[\frac{\frac{\text{kg} \cdot \text{m}}{\text{sec}^2} \cdot \text{m}^2}{\text{kg} \cdot \text{kg}} \right] = \frac{\text{kg} \cdot \text{m}^3}{\text{sec}^2 \cdot \text{kg}^2}$$

$$[G] = [kg^{-1} m^3 \text{Sec}^2]$$

$$[G] = [M^{-1} L^3 T^2]$$

Numerical problem 7: Show that

(a) $KE = \frac{1}{2} m v^2$ (b) $PE = mgh$ are dimensionally correct.

(a) Given that: $KE = \frac{1}{2} m v^2$

$$\text{Taking L.H.S } [KE] = [\text{Joule}]$$

$$= [Nm]$$

$$= \left[\frac{\text{kg} \cdot \text{m}}{\text{sec}^2} \cdot \text{m} \right] = \left[\frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2} \right]$$

$$[KE] = [ML^2T^{-2}] \text{----- (i)}$$

Taking R.H.S
$$\left[\frac{1}{2}mv^2\right] = \left[kg \left(\frac{m}{sec}\right)^2\right]$$

$$= \left[kg \frac{m^2}{sec^2}\right]$$

$$\left[\frac{1}{2}mv^2\right] = [ML^2T^{-2}] \text{----- (ii)}$$

From eq (i) & (ii) we get that $KE = \frac{1}{2}mv^2$ is dimensionally correct.

(b) Given that: $PE = mgh$

To show it is dimensionally correct

Taking L.H.S $[PE] = [Joule]$

$$= Nm = \left[kg \frac{m}{sec^2} \cdot m\right]$$

$$= [kgm^2 sec^{-2}]$$

$$= [ML^2T^{-2}] \text{----- (i)}$$

Taking R.H.S

$$[mgh] = \left[kg \frac{m}{sec^2} \cdot m\right]$$

$$= [kgm^2 sec^{-2}]$$

$$= [ML^2T^{-2}] \text{----- (ii)}$$

From eq (i) & (ii) we get that $PE = mgh$ is dimensionally correct.

Unit # 2

Vectors and EquilibriumExercise Long Questions

Q. 1: How are vectors added and subtracted geometrically?

Ans: Addition of Vectors:

The geometric process in which two or more than two vectors are replaced by a single vector is called addition of vectors.

Resultant Vector:

The vector obtained as a result of addition of vectors is called resultant vector, it is represented by $\vec{R}$ or $\vec{C}$.

The length of resultant vector may or may not be equal to the added vectors but the effect of the resultant vector must be equal to all the added vectors.

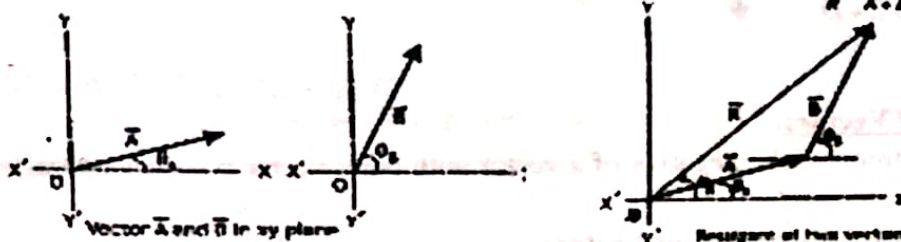
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \dots\dots\dots$$

Head to Tail Rule of Vector's Addition:

The rule in which the tail of the second vector (lagging) is joined to the head of the first (leading) vector and so on is called head-to-tail rule of vector addition.

Steps to add two or more vectors by head-to-tail rules:

- i) Draw first vector according to a suitable and common scale in the given direction.
- ii) Put the tail of second vector on the head of the first vector according to the scale in the given direction.
- iii) In case of more vectors, the tail of the 3rd vector is joined with the head of second vector and so on.
- iv) Draw the resultant vector by joining the tail of first vector with the head of the last vector.
- v) Measure the length of resultant vector and convert it back according to adopted scale.
- vi) Measure the angle of the resultant vector to determine its direction.

Graphically:Commutative Property of Vector Addition:

If the order (ترتيب) of given vectors is changed in adding vectors but the resultant vector remains unaffected, this property of vector addition is called commutative property with respect to addition.

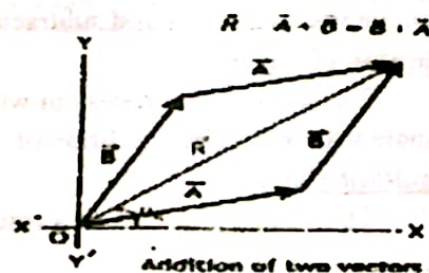
Mathematically:

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Figuratively:

Let two vectors $\vec{A}$ and $\vec{B}$ are given then
 $\vec{A} + \vec{B} = \vec{B} + \vec{A} = \vec{R}$

One can see from the figure that the resultant vector does not change whether $\vec{A}$ is added to $\vec{B}$ or $\vec{B}$ is added to $\vec{A}$.

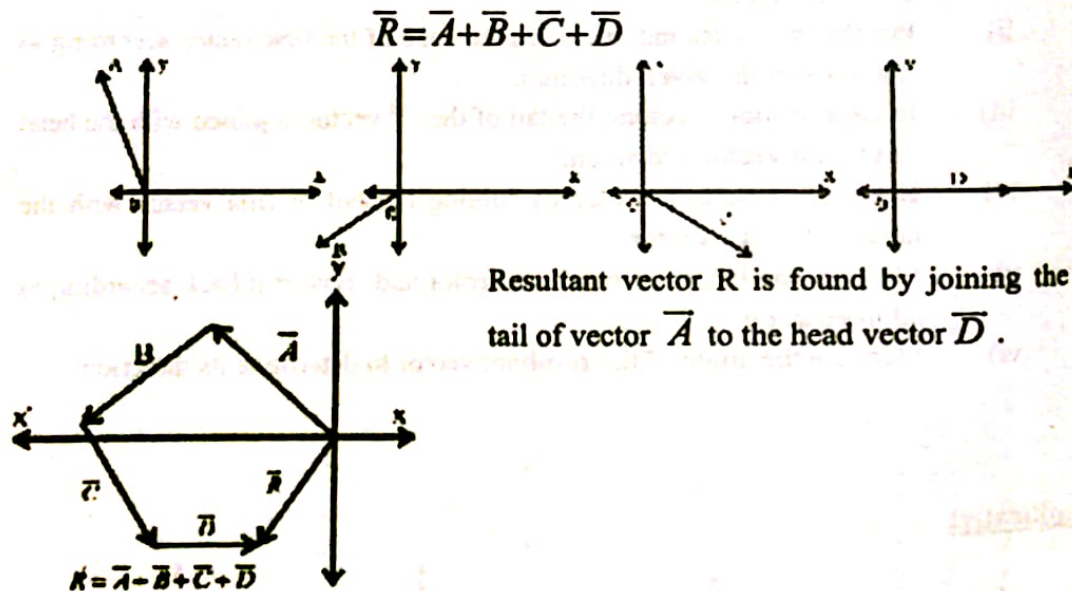


Addition of More Than Two Vectors:

In case of more than two vectors, the tail of the second vector is joined with the head of first vector, similarly, the tail of the 3rd vector is joined with the head of second vector and this process of adding the vectors is continued till last vector is added. The resultant vector R is obtained by joining the tail of first vector to the head of last vector.

Example:

Let, $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ four vectors are given their resultant is found by the rule mentioned above.



Subtraction of Vectors:

The addition of the negative of a vector with other vector is called subtraction of vectors.

Steps to Perform Subtraction of Vectors:

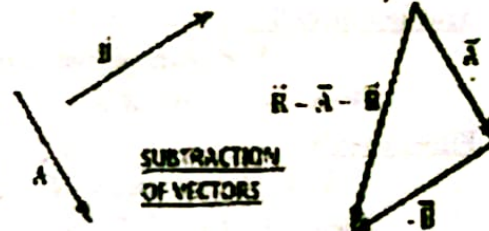
- i) Draw the vectors according common scale.

- ii) Find the negative of that vector which is to be subtracted e.g. $\vec{B} = -\vec{B}$.
- iii) Add the vector $-\vec{B}$ according to head to tail rule.
- iv) Find the resultant $\vec{R}$ by joining the tail of first vector $\vec{A}$ to the head of second vector $-\vec{B}$.

Figuratively:

$$\vec{R} = \vec{A} - \vec{B} \text{ or } \vec{R} = \vec{A} + (-\vec{B}).$$

When a vector is multiplied by (-ve) in other words when the negative of a vector is taken, the direction of the negative vector reverses or changes by 180° and its magnitude does not change.



Q. 2: If a vector is multiplied by a positive scalar, how is it related to the original vector? What if the scalar is zero? Negative?

Ans: Multiplication of Vector by Positive Scalar:

If K is a positive scalar (number, بندم) and $\vec{A}$ is vector, and vector $\vec{A}$ is multiplied by K , consequently (نتیجتاً) the magnitude of vector $\vec{A}$ will change K times and its direction will remain unchanged.

Mathematically:

If $K > 0$ (positive scalar)

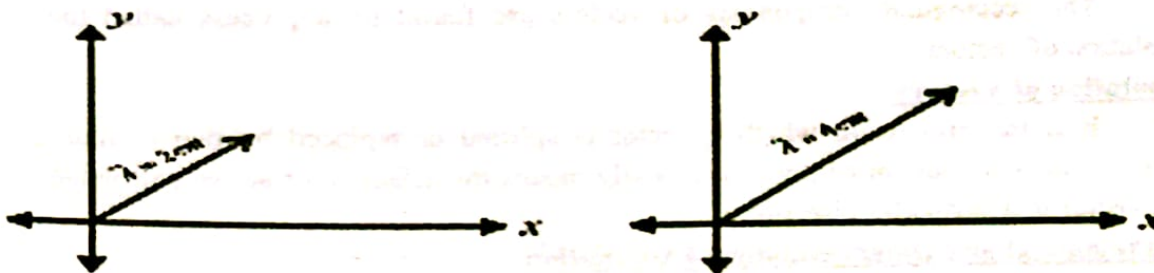
$$\text{Then } +K \cdot \vec{A} = K\vec{A}$$

Figuratively:

If $K = +2$

$$\text{Then } K\vec{A} = 2\vec{A}$$

If $\vec{A}$ is given as



Multiplication of a Vector by Zero:

If a vector is multiplied by zero, the resultant vector will be a Null vector.

Mathematically:

If $K = 0$ (zero scalar)

$$KA = 0A = \vec{0}$$

Figuratively:

It is impossible to draw the Null vector because it would have arbitrary direction.

Multiplication of a Vector by Negative Scalar:

If a vector is multiplied by a negative scalar (منفی بندس) the magnitude of the vector would change scalar times vector and the direction would be reversed or changed by 180° .

Mathematically:

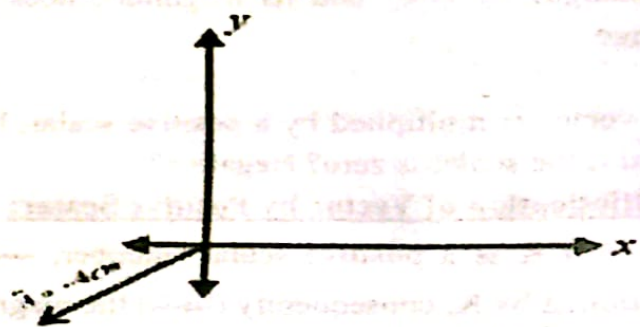
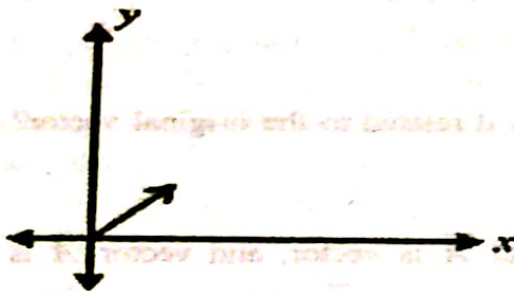
If $K < 0$ (negative scalar)

Then $-K \vec{A} = -K \vec{A}$

Figuratively:

If $K = -2$ and vector $\vec{A} = 2\text{cm}$ as shown.

Then $-K \vec{A} = -2(2\text{cm}) = -4\text{cm}$ (opposite direction)



Q. 3: What are rectangular components of a vector? How rectangular components are used to represent a vector?

OR

Explain Resolution of vectors.

Ans: **Rectangular Components of a Vector:**

The components of a vector which are mutually perpendicular to each other are called rectangular components of that vector.

The rectangular components of vectors are found by a process called the resolution of vectors.

Resolution of Vectors:

It is the process in which a vector is splitted or replaced by two or more components. Component of a vector actually means the effective or active part/value of a vector in a particular direction.

Mathematical and Diagrammatical Explanation:

Consider a vector $\vec{A}$ represented by line $\overline{OP}$ in Cartesian coordinate system, as shown in figure.

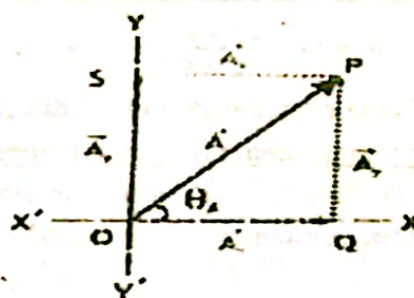
Draw perpendiculars from point P on x and y-axis $\overline{OQ}$ represents the x-component or horizontal component, while $\overline{PQ}$ represent the y-component or vertical component of vector $\overline{A}$. Vector has been splitted in two parts i.e.

$$\overline{A} = \overline{A}_x + \overline{A}_y$$

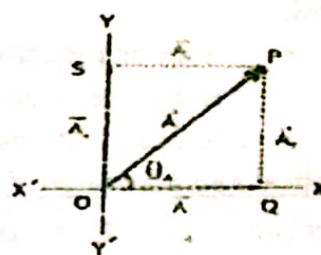
OR $\overline{A} = A_x \hat{i} + A_y \hat{j}$

Representation of a Vector in Terms of its Components:

Consider a vector $\overline{A}$ having x and y-components as shown in figure let $\overline{OQ}(A_x)$ and $\overline{QP}(\overline{A}_y)$ represent the sides of a right angle triangle. From trigonometry.



Vector represented in term of its components.



Vector represented in term of its components.

$$\cos \theta = \frac{\text{Base}}{\text{hyp}} = \frac{A_x}{A} \quad \text{OR} \quad A_x = A \cos \theta \quad \text{-----(i)}$$

$$\sin \theta = \frac{\text{perp}}{\text{hyp}} = \frac{A_y}{A} \quad \text{OR} \quad A_y = A \sin \theta \quad \text{-----(ii)}$$

As $\overline{A} = A_x \hat{i} + A_y \hat{j}$ Put (i) and (ii) in this equation

$$\overline{A} = A \cos \theta \hat{i} + A \sin \theta \hat{j} \quad \text{-----(iii)}$$

Equation (iii) show the representation of a vector in terms of its components.

Vector in its Components Form:

Applying pythagoras theorem to triangle ΔOPQ one can write.

$$(\text{hyp})^2 = (\text{Base})^2 + (\text{perp})^2$$

$$A^2 = A_x^2 + A_y^2$$

OR $|A| = \sqrt{A_x^2 + A_y^2} \quad \text{-----(iv)}$

This is the equation to find the magnitude of the original vector from its components.

Direction of Original Vector:

The direction of the original (splitted) vector can be found from its components by the use of trigonometry.

$$\tan \theta = \frac{\text{perp}}{\text{Base}} = \frac{A_y}{A_x}$$

OR $\theta = \tan^{-1} \left(\frac{Ay}{Ax} \right) \text{-----} (v)$

This is the equation to find the direction of vector from its components.

Rectangular Components in Three Dimensional Coordinate System or Space:

In three dimensional coordinate system or space a vector can be splitted or resolved in three components i.e. x, y and z-components, in this case

$$|A| = \sqrt{Ax^2 + Ay^2 + Az^2}$$

Q. 4: Explain the addition of vectors by rectangular components.

Ans: Definition:

The method of addition in which the rectangular components of a vector are added to get the resultant vector is called addition of vectors by rectangular components.

- It is the analytical method of vectors addition.
- It is the reverse process of resolution of vectors.
- It is an accurate and precise method of vectors addition.

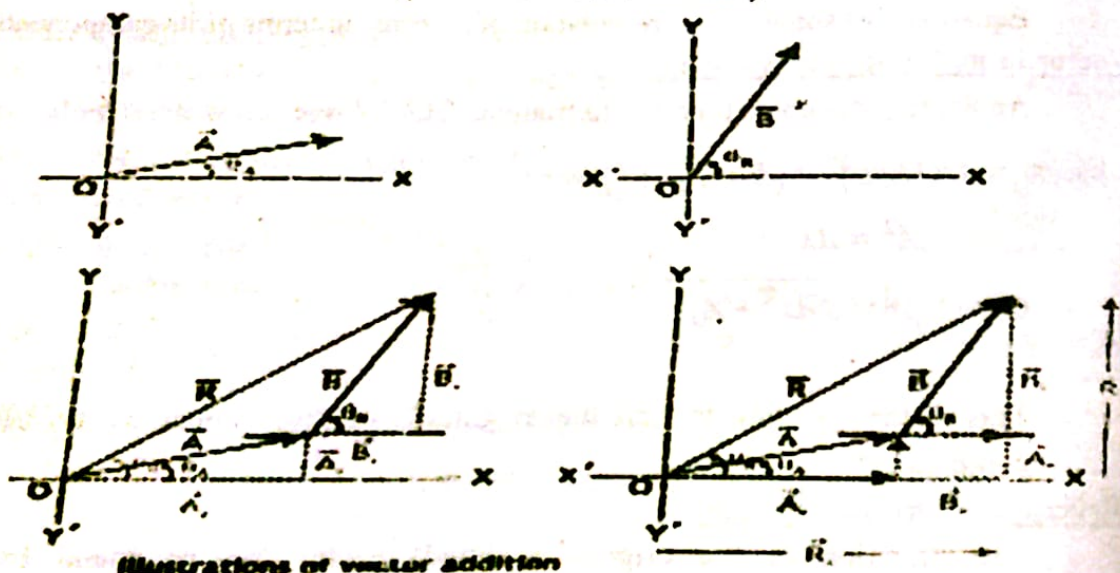
Mathematical and Figurative Explanation:

Let two vectors $\vec{A}$ and $\vec{B}$ are given as shown.

Steps for Addition of Vectors by Rectangular Components:

- Draw the vectors according to a suitable and common scale, in the given direction.
- Add the vectors according to head-to-tail rule.
- Find the horizontal and vertical components.
- Find the direction of resultant vector according to relation.

$$\theta = \tan^{-1} \left(\frac{Ay + By + Cy + \dots}{Ax + Bx + Cx + \dots} \right)$$



Illustrations of vector addition

The components of vector $\vec{R}$ (resultant) as shown above are given below.
It can be seen from the above figure that,

$$Rx = Ax + Bx \text{-----} (i)$$

$$R_y = A_y + B_y \text{-----(ii)}$$

Also $\vec{R} = R_x \hat{i} + R_y \hat{j} \text{-----(iii)}$

Put (i) and (ii) in (iii)

$$\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

For more than two vectors

$$\vec{R} = (A_x + B_x + C_x + \dots) \hat{i} + (A_y + B_y + C_y + \dots) \hat{j}$$

Magnitude of R:

For magnitude use the relation

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

OR $|\vec{R}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$

Direction of R:

For direction of R use

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

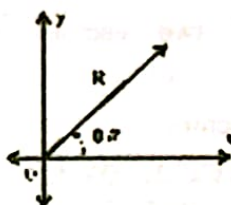
$$\theta = \tan^{-1} \left(\frac{A_y + B_y}{A_x + B_x} \right)$$

For more than two vectors

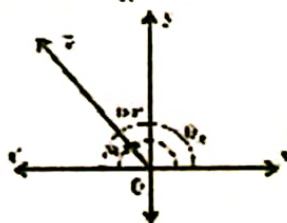
$$\theta = \tan^{-1} \left(\frac{A_y + B_y + C_y + \dots}{A_x + B_x + C_x + \dots} \right)$$

Determination of angle "θ":

- i) If R_x and R_y both are positive then θ lies in the first quadrant and can be calculated by using $\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$

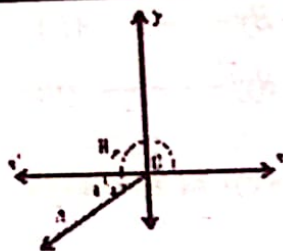


- ii) If R_x is negative while R_y is positive θ lies in the second quadrant and can be determined by $\theta_R = 180^\circ - \theta$.



$$\theta + \theta_R = 180^\circ \quad \text{OR} \quad \theta_R = 180^\circ - \theta$$

- iii) If R_x and R_y both are negative θ lies in the third quadrant and can be determined by $\theta = 180^\circ + \theta$



- iv) If R_x is positive and R_y negative θ lies in the fourth quadrant and can be found by $\theta_R = 360^\circ - \theta$



Q. 5: Define the dot product of two vectors. What geometric interpretation the dot product have? Give examples.

Ans: Dot Product (Scalar Product):

When a vector is multiplied by a vector and the resultant quantity is a scalar it is called dot product or scalar product.

Reason for the Names Dot and Scalar Product:

It is called dot product due to the reason that it is represented by putting a dot between vectors.

It is called scalar product due to the reason that it gives a scalar quantity as a product.

Mathematically:

The dot (scalar) product of two vectors $\vec{A}$ and $\vec{B}$ is defined as $\vec{A} \cdot \vec{B} = AB \cos \theta$

Where θ is the angle between vectors.

Geometrical Interpretation of Dot (Scalar) Product:

Dot product of two vectors is the product of one vector with that component of second vector which is in direction of that vector, and vice versa i.e.

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A(B \cos \theta) \text{----- (i)}$$

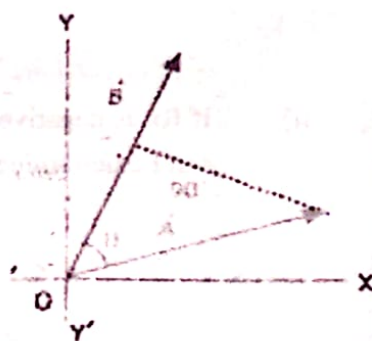


Illustration of scalar product

$$\Rightarrow \vec{A} \cdot \vec{B} = (\text{vector } \vec{A}) \cdot (\text{magnitude of component of vector } \vec{B} \text{ in the direction of } \vec{A})$$

$$\text{Also } \vec{B} \cdot \vec{A} = (A \cos \theta) B \text{----- (ii)}$$

$$\vec{B} \cdot \vec{A} = (\text{magnitude of the component of vector A in the direction of } \vec{B}) (\text{magnitude of } \vec{B})$$

Equation (i) and (ii) show that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Which proves commutative law or property for dot product.

Figuratively:

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$A \cdot B = A(B \cos \theta)$$

Can be figuratively shown as:

$\vec{B} \cdot \vec{A} = B(A \cos \theta)$ can also be represented.

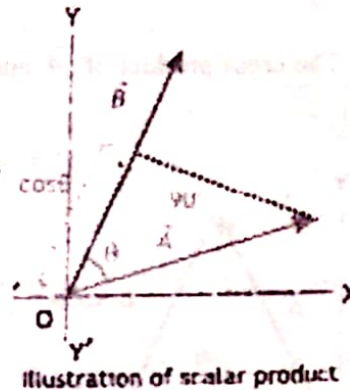


Illustration of scalar product

Examples of Vectors Product:

- i) $work = \vec{F} \cdot \vec{S}$
- ii) $Electric\ Flux = \vec{E} \cdot \vec{A}$
- iii) $Power = \vec{F} \cdot \vec{V}$
- iv) $Magnetic\ Flux = \vec{B} \cdot \vec{A}$

the cross product (vector product) of two vectors. What geometric interpretation the does the cross product have? Give examples.

Ans: Cross Product (Vector Product):

When a vector is multiplied by another vector and the resultant quantity is a vector it is called cross product or vector product.

vector $\times$ vector = vector

Reason for the Names Cross and Vector Product:

It is called cross product because it is represented by putting a cross between vectors.

It is called vector product because it gives a vector quantity as a result.

Mathematically:

The cross (vector) product of two vectors $\vec{A}$ and $\vec{B}$ is defined as $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$.

Where θ is the smaller of angle between $\vec{A}$ and $\vec{B}$, and $\hat{n}$ is a normal vector perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$.

Geometric Interpretation of Vector Product:

Cross (vector) product of two vectors is the product of magnitude of one vector and the magnitude of the component of second vector which is perpendicular to that vector, i.e.

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$$\vec{A} \times \vec{B} = A(B \sin \theta) \hat{n}$$

$$\vec{A} \times \vec{B} = (\text{Magnitude of } \vec{A}) (\text{magnitude of the component of } \vec{B} \perp \text{ to } \vec{A})$$

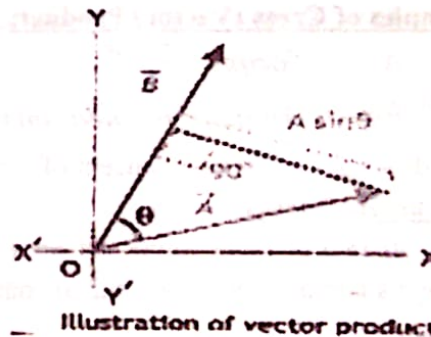
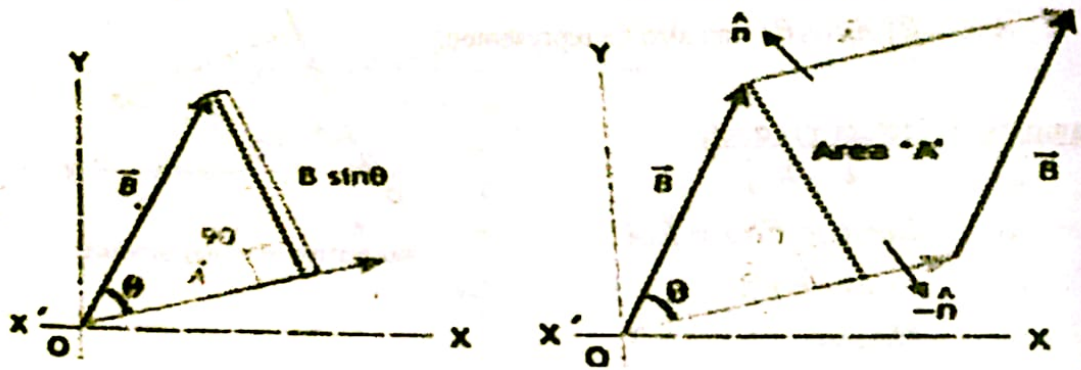


Illustration of vector product

The magnitude $AB \sin \theta$ is actually the area of the plane formed by $\vec{A}$ and $\vec{B}$. The direction of the resultant vector is always perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$.

Figuratively:

The cross product of $\vec{A}$ and $\vec{B}$ is vertically shown as:



Right Hand Rule or Thumb Rule:

This rule states that rotate the fingers of your right hand from first vector towards second vector the stretched (بسط) thumb will give the direction of resultant vector.

Anti-Commutative Property/Law of Vector Product:

If the order of vectors $\vec{A}$ and $\vec{B}$ in vector product is changed the direction of the resultant vector is reversed or changes by 180° , this property of vector product is called anti-commutative property of vector product i.e.

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} \text{----- (i)}$$

$$\text{But } \vec{B} \times \vec{A} = AB \sin \theta (-\hat{n})$$

$$\text{OR } \vec{B} \times \vec{A} = -AB \sin \theta \hat{n} \text{----- (ii)}$$

Equations (i) and (ii) show that

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\text{OR } \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Examples of Cross (Vector) Product:

$$\text{i) } \text{Torque} = \vec{r} \times \vec{F}$$

$$\text{ii) } \text{Angular momentum} = \vec{r} \times \vec{P}$$

Q. 11: Elucidate (روشنی ڈالیں) the concept of concurrent forces.

Ans: Concurrent Forces:

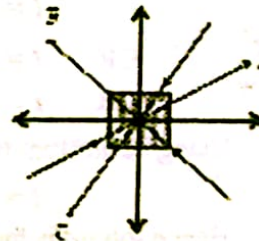
If two or more forces act on a body and the line of action these forces pass through a common point, are called concurrent forces.

Example:

Let a block is acted upon by three forces as shown in figure. The line of action of these forces pass through the central point of the block which is a common point to all forces, hence these are regarded as concurrent forces.

Diagrammatically:

These forces are diagrammatically shown as:



Equilibrant Force:

The effectively equivalent force which balances the concurrent forces is called equilibrant force.

Q. 7: Define torque. Show that torque is the vector product of force and position vector.

Ans: **Torque or Moment of Force:**

The effectiveness of a force to produce rotational acceleration in a body.

OR

The turning effect of force is called torque.

OR

The cross (vector) product of force and position vector is called torque.

Nature of Torque:

Torque is a vector quantity and its direction is found by right hand rule.

Unit of Torque:

The unit of torque is Nm, which should not be considered as joule.

Dimensions of Torque:

The dimensions of torque are:

$$[\text{torque}] = [\vec{r} \times \vec{F}]$$

$$[\vec{\tau}] = [rma]$$

$$[\vec{\tau}] = \left[rm \frac{v}{t} \right]$$

$$[\vec{\tau}] = \left[ML \frac{T^{-1}}{T} \right]$$

$$[\vec{\tau}] = [ML^2T^{-2}]$$

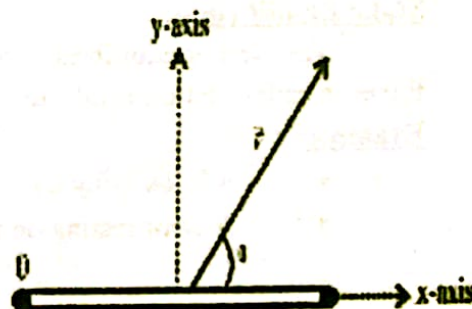
Positive and Negative Torque:

Clockwise torque is taken as negative while anticlockwise torque is taken as positive.

To Show That Torque is the Vector Product of Force and Position Vector:

Consider a force $\vec{F}$ is applied at a point of a rod pivoted at point "O" as shown in figure.

The force $\vec{F}$ has two components i.e. F_x or x-component which acts along x-axis and does not rotate or produce torque and F_y or y-component which acts along y-axis and is that component which rotates the



rod and produce torque, this component is perpendicular to position vector.

Mathematically it can be written as:

$$t = rFy$$

OR $t = rF \sin \theta$

OR $t = r(\text{magnitude of } \perp \text{ component of force})$

Using definition of vector product

$$\vec{t} = \vec{r} \times \vec{F}$$

Hence torque is the vector product of force and position vector.

Factors on Which Torque Depends:

- Magnitude of applied force $\vec{F}$
- Magnitude (length) of position vector $\vec{r}$
- Angle between position vector and force

Force and position vector directly affect magnitude of torque. For a given force and position vector the torque then depends upon angle between force ($\vec{F}$) and position vector ($\vec{r}$). Torque is maximum if $\theta = 90^\circ, 270^\circ$, torque is zero if $\theta = 0^\circ, 180^\circ, 360^\circ$.

Q. 8: What is mechanical equilibrium? Explain different types of equilibrium.

Ans: Physical Definition:

The mechanical state of a body under the action of forces and torques in which there is no change in translational and rotational motion is called mechanical equilibrium.

OR

The state of rest or uniform motion of a body is called mechanical equilibrium.

Mathematical Definition:

The non-accelerated ($\vec{a} = 0$) mechanical state of an object is called mechanical equilibrium.

Types of Equilibrium:

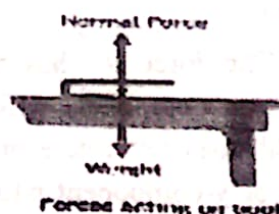
There are the following types of equilibrium.

- Static equilibrium
 - Dynamic equilibrium
 - Dynamic translation equilibrium
 - Dynamic rotational equilibrium
- i) Static Equilibrium:

The type of equilibrium in which the body is at rest under the action of forces is called static equilibrium.

Example:

- A book lying on table.
- A chair resting on the floor.

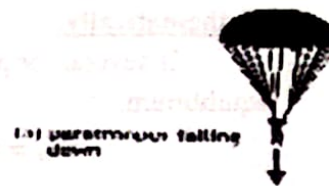


ii) **Dynamic Equilibrium:**

The type of equilibrium in which a body moves with uniform velocity under the action of forces, is called dynamic equilibrium.

Example:

- Falling of paratrooper
- Motion of swift (bird)
- Motion of CD disc
- Motion of grind stone

**Types of Dynamic Equilibrium**

There are two types of dynamic equilibrium.

(a) **Dynamic Translational Equilibrium:**

The type of dynamic equilibrium in which a body moves with uniform linear (translational) velocity under the action of forces is called dynamic translational equilibrium.

Example:

- Falling of paratrooper
- Motion of swift

(b) **Dynamic Rotational Equilibrium:**

The type of dynamic equilibrium in which a body moves with uniform angular (rotational) velocity is called dynamic rotational equilibrium.

Example:

- Compact Disc (CD) motion
- Motion of grind stone

Q. 9: What type of equilibrium is guaranteed by each condition of equilibrium.

Ans: There are the following two conditions of equilibrium:

- i) First condition of equilibrium
- ii) Second condition of equilibrium

i) **First Condition of Equilibrium:**

If the vector sum of all forces acting on a body is zero the body is said to be in a state of translational equilibrium.

Mathematically:

According to the first condition

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = 0$$

$$\text{OR} \quad \vec{F}_{net} = \sum_{i=1}^{i=n} \vec{F}_i = 0$$

For Coplanar Forces:

If the forces are coplanar (lying in one plane), the sum of the x-components and y-components of forces must be equal to zero.

$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j} = \vec{0}$$

First condition of equilibrium guarantees translational equilibrium.

ii). **Second Condition of Equilibrium:**

If the vector sum of all torque acting on a body is zero the body is used to be in a state of equilibrium, is called second condition of equilibrium.

Mathematically:

If several torques act on a body then, according to second condition of equilibrium.

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots = 0$$

$$\text{OR } \vec{\tau}_{net} = \sum_{i=1}^n \vec{\tau}_i = \vec{0}$$

In other words

$$\sum \vec{\tau}_{\text{clockwise}} = -\sum \vec{\tau}_{\text{anticlockwise}}$$

(Anticlockwise torque is negative)

$$\Rightarrow \vec{\tau}_{net} = \sum \vec{\tau}_{\text{clockwise}} = -\sum \vec{\tau}_{\text{anticlockwise}} = 0$$

Second condition of equilibrium guarantees rotational equilibrium.

Complete Mechanical Equilibrium:

If only the first condition of equilibrium is satisfied it guarantees translational equilibrium and does not guarantee rotational equilibrium, and if the only second condition of equilibrium is satisfied it guarantees rotational equilibrium and does not guarantee translational equilibrium.

So for a complete equilibrium both the conditions of equilibrium must simultaneously be satisfied.

Non-exercise Long Questions

Q. 1: Explain the concept of vectors, highlight representation of vectors.

Ans: **Vectors:**

Those physical quantities which need magnitude as well as direction for their expression or specification (وضاحت) are called vectors or directed quantities.

Example:

Force, velocity, acceleration etc.

Concept of Vectors:

If a car need to be pushed to get started, it will need a force to be applied. But if the force is applied in a direction opposite to the direction in which the driver has applied gears, it would rather waste the push of other men.

Therefore it is needed that the force be applied in the desired and required direction.

Operation (عمل) of Vectors:

The mathematical operations i.e. addition, subtraction, multiplication and resolution (division) are different from the rules of arithmetics. Vectors have their own algebra called vector algebra.

Point to PONDER:

A physical quantity may need direction but if it does not follow rules of vector algebra this quantity would be considered as scalar.

Example:

Electric current is a scalar although it needs direction.

Representation of Vectors:

Vectors are represented in two ways:

- i) Symbolic Representation
- ii) Graphical Representation

Symbolic Representation (علامتی اظہار):

Symbolically vectors are represented by bold-face letters (موٹے حروف) e.g. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{a}$, $\mathbf{b}$, or by simple face letters with an arrow (تیر) above or below the letter e.g.

$(\vec{A}, \underline{A}, \vec{B}, \underline{B}, \vec{a}, \underline{a})$ etc.

Representation of Magnitude of Vector:

The magnitude of a vector is either represented in Italics or writing the vector inside two vertically parallel lines (absolute or determinant symbol), e.g. $|\vec{A}|$ or $|\underline{B}|$

Representation of Direction of Vector:

The direction of a vector is represented by putting a cap symbol over the letter e.g. the direction of vector $\vec{A}$ will be represented by $\hat{A}$.

General Relation for Vectors:

The general formula for vectors is given as $\vec{A} = |\vec{A}| \hat{A}$ in which

$|\vec{A}| = \text{magnitude}$, $\hat{A} = \text{direction}$,

Graphical Representation of Vectors:

Graphically a vector is represented by a line segment with an arrow. The length of the line segment should be equal to the magnitude of vector while the direction of vector would be represented by the arrow head.

Tail of Vector:

The starting point of a vector is called tail of vector.

Head of Vector:

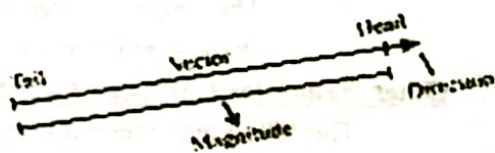
The end point of a vector is called head of vector.

Geometric Vectors:

If a vector(s) is represented without reference to certain axes is called geometric vector.

Algebraic Vectors:

Those vectors which are represented with reference to a system of axis or plane are called algebraic vectors.



Q. 2: Define Cartesian coordinate system. Discuss representation of vectors in Cartesian coordinate system (plane) and space.

straight lines intersecting at a point, which is used to present and localize vectors is called Cartesian coordinate system.

X-Axis:

The horizontal line is called x-axis. Positive x-axis lie to the right of the origin while negative to the left of the origin.

Y-Axis:

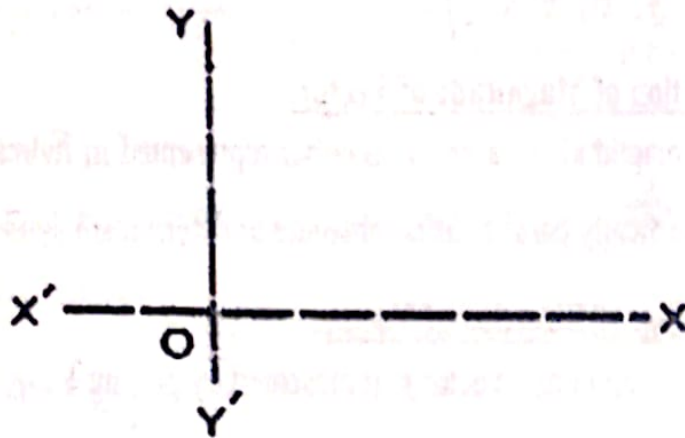
The line which is vertical is called y-axis. Positive y-axis lie above the origin while negative y-axis lies below the origin.

Origin:

The point of intersection of the x-axis and y-axis is called origin of the system.

Diagrammatically:

The Cartesian coordinate system is diagrammatically shown below:



Cartesian Coordinate System

The Cartesian coordinate system has four equal parts called quadrants.

Steps to represent a vector in Coordinate System:

- i) Draw coordinate system.
- ii) Select a suitable scale.
- iii) Draw a line in the given direction.
- iv) Cut the line equal to the magnitude of vector according to the selected scale.
- v) Put an arrow in the direction of the vector.
- vi) The angle is taken with reference, to the positive x-axis in the anti-clock wise direction.

Other Names for Cartesian Coordinate System:

The following other names are also used for Cartesian coordinate system.

- i) Plane
- ii) Two dimensional coordinate system
- iii) Reference frame

Coordinate System

direction of vector with respect to x, y and z-axis respectively.

Space is also called three dimensional coordinate system.

Q. 3: Discuss the concept of unit vectors.

Ans: The vector whose magnitude is unity (one=1) and is used to represent the direction of a vector is called unit vector(s). These are dimensionless and unit less vectors.

General Representation of Unit Vectors:

A unit vector is generally represented by a letter with a cap or hat over it, e.g. the unit vector for $\vec{A}$ is $\hat{A}$.

Formula to Find Unit Vector:

The general formula for vectors is given as

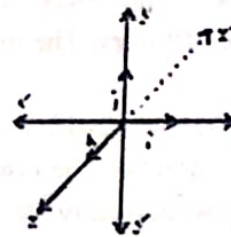
$$\vec{A} = |\vec{A}| \cdot \hat{A}$$

or

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

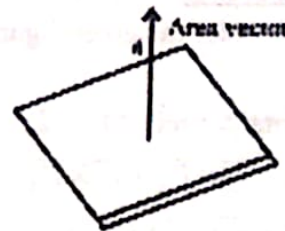
$\hat{i}$, $\hat{j}$, $\hat{k}$ Unit Vectors:

$\hat{i}$, $\hat{j}$, $\hat{k}$ Unit vectors are used to represent the direction of x-, y- and z-axes respectively. These vectors do not change the magnitude of other vectors and even scalars.



Normal (عمودی) Unit Vectors:

The direction of certain surfaces or areas is represented by a vector which is perpendicular to the surface or plane is called normal (vertical) vector. Its magnitude as, already mentioned is one. It is also called as area vector.



Q. 4: Define Null vector and subtraction of vectors.

Ans: **Null Vectors:**

The vector having zero magnitude and some arbitrary direction, is called Null vector.

Representation:-

Null vector is usually denoted by $\vec{O}$.

Concept of Null Vector:

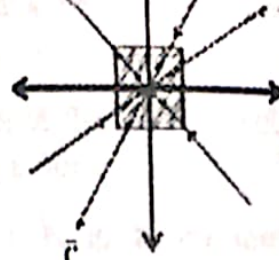
If two equal and oppositely directed forces acting at a single point are added i.e. $\vec{F} - \vec{F} = \vec{O}$.

Null vectors are obtained from the operation of vectors.

The line of action of these forces pass through the central point of the block which is a common point to all forces, hence these are regarded as concurrent forces.

Diagrammatically:

These forces are diagrammatically shown as:



Equilibrant Force:

The effectively equivalent force which balances the concurrent forces is called equilibrant force.

Q. 6: Define moment of couple. Give an example.

Ans: Two equal and oppositely directed forces separated by perpendicular distance are called couple of forces. The moment produced by couple is called moment of couple.

Example:

A driver, while driving applies forces on the steering wheel. One hand applies force in one direction the other hand in opposite direction it makes him able to turn the steering wheel easily. If the applied forces are equal in magnitude it is called couple.

Mathematically:

Consider the given figure:

$$\text{Total torque} = t_A + t_B$$

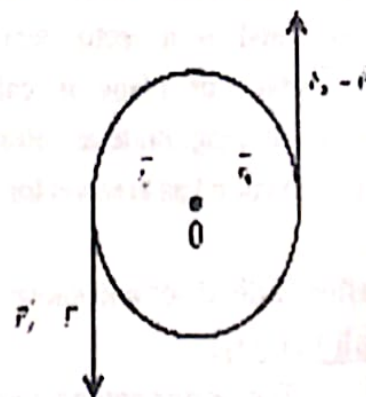
$$\vec{\tau} = \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B$$

$$\vec{\tau} = \vec{r}_A \times (-\vec{F}) + \vec{r}_B \times \vec{F}$$

$$\because F_A = F_B = F$$

$$\vec{\tau} = (\vec{r}_B - \vec{r}_A) \times \vec{F}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$



Direction of Couple Moment:

The direction of moment of couple is determined by right rule.

NOTE:

The term moment and movement are completely different terms and carry different physical meanings.

Q. 7: How cranes are able to lift heavy loads without toppling?

Ans: Cranes are able to lift heavy loads without toppling is due to counter-torque (weight).

Explanation:

Mobile cranes lift or lower (اتارنا) and transport loads. It works on the principles of lever (machine). The longer end (boom) positions (ركبنا) the object at the desired location. Due to larger length of boom a large torque produces which increases the chances of toppling over the mobile crane. This danger is overcome by a counter weight at the other side to produce counter torque. Consequently the mobile crane, can lift heavy load without toppling.

$$L_{\text{boom}} = L_{\text{counter torque}}$$

Short Questions & Answers

Q. 1: Is it possible to add three vectors of equal magnitude but different directions to get a null vector? Illustrate with a diagram.

Ans. Yes, it is possible to add three vectors of equal magnitude, but different directions to get a Null vector.

Example:

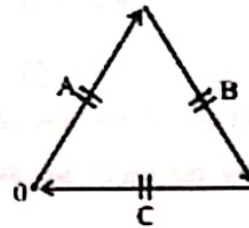
Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ of equal magnitudes i.e. ($A=B=C$) and different directions.

Add them to form an equilateral triangle.

Diagram:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{R} = 0 \text{ [Null vector]}$$



Null Vector:

Null vector is a vector having zero magnitude and arbitrary direction. Vector "R" in the above diagram is a Null vector.

Conclusion:

Three vectors of equal magnitudes but different directions can be added to get a Null vector in the form of an equilateral triangle

Q. 2: The magnitudes of three vectors are 2m, 3m and 5m respectively. The directions are at your disposal. Can these three vectors be added to yield zero? Illustrate with a diagram.

Ans. Yes, they can be added to yield zero.

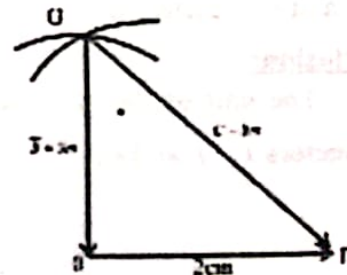
Diagrammatic Illustration:

- The magnitudes of three vectors are $\vec{A} = 2m$, $\vec{B} = 3m$ and $\vec{C} = 5m$, respectively.
- Their directions are random.
- Choose a scale

Let $1m = 1cm$ then

$$\vec{A} = 2m = 2cm$$

$$\vec{B} = 3m = 3cm$$



$$\overline{C} = 5m = 5cm$$

- Draw vector $\overline{A} = 2cm$, from "0" to "P"
- Construct an arc at a distance of 3cm from "0" with a compass.
- Construct another arc at a distance of 5cm from "0" which intersect the first arc at a point (say "G")
- Draw a vector from point "P" to "G" $\overline{PG} = 5cm$ which is vector $\overline{C} = 5cm$.
- Draw a vector from point "G" to "0" such that $\overline{GP} = 3cm$ which is vector $\overline{B} = 3cm$.
- It is clear from the diagram that result of $\overline{A} + \overline{B} + \overline{C}$ is zero.

Conclusion:

Three vectors of magnitudes $A = 2cm$, $B = 3m$ and $C = 5cm$ can be added to yield zero.

Q.3: What units are associated with the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$?

Ans. Generally no units are associated with the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ as,

$$\begin{aligned}\overline{A} &= |A| \hat{A} \\ \Rightarrow \hat{A} &= \frac{\overline{A}}{|A|}\end{aligned}$$

The units are cancelled out thus unit vectors are dimension less and unit less vectors.

Generally no units are associated with unit vector $\hat{i}$, $\hat{j}$ and $\hat{k}$. But under certain conditions one may associate units with unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$. In that case the units associated with $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the units of the physical quantity under consideration.

Conclusion:

The units associated with the unit vectors changes from case to case.

Example:

If somebody is talking about force. "F" e.g. $\overline{F} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ Then in this case the unit associated with $\hat{i}$, $\hat{j}$ and $\hat{k}$ is newton "N" which is the unit of force.

Similarly:

In case of displacement their unit is meter "m". $\overline{d} = 3\hat{i} + 5\hat{j} + 2\hat{k}$ Unit of $\hat{i}$, $\hat{j}$ and $\hat{k}$ is meter "m".

Conclusion:

The unit of the physical quantity under consideration is associated with the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$.

Q. 4: Can a scalar product of two vectors be negative? Provide a proof and give an example.

Ans. Yes, the scalar product of two vectors can be negative, if the angle between them is more than 90° and less than 180° angle. i.e. $90^\circ < \theta < 180^\circ$

Proof:

The scalar product of two vectors A & B is defined as:

$$\vec{A} \cdot \vec{B} = AB \cos \theta \text{ --- (i)}$$

$\cos \theta$ is negative in the range 90° to 270° ($90^\circ < \theta \leq 180^\circ$)

If $\theta = 180^\circ$

$$\Rightarrow \vec{A} \cdot \vec{B} = AB \cos(180^\circ)$$

$$\because \cos 180^\circ = -1$$

$$\Rightarrow \vec{A} \cdot \vec{B} = AB(-1)$$

$$\Rightarrow \vec{A} \cdot \vec{B} = -AB$$

Thus the scalar product of two vectors can be negative.

Example:

The scalar product of frictional force and displacement, because frictional force always acts oppositely to the direction of motion (displacement).

Q. 5: $\vec{A}$ and $\vec{B}$ Are two nonzero vectors. How can their scalar product be zero? And how can their vector product be zero?

Ans. $\vec{A}$ and $\vec{B}$ are two nonzero vectors. Their scalar and vector product can be zero in the following ways.

Zero Scalar Product:

The scalar/dot product of two nonzero vectors $\vec{A}$ and $\vec{B}$ vanishes (ختم ہو جاتا) when the angle between them is 90° . i.e. $\vec{A}$ and $\vec{B}$ are mutually perpendicular ($A \perp B$).

Mathematically:

The scalar product of $\vec{A}$ and $\vec{B}$ is defined as:

$$\vec{A} \cdot \vec{B} = AB \cos \theta \text{ --- (i)}$$

- Where "A" and "B" are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$
- " θ " is the angle between the vectors $\vec{A}$ and $\vec{B}$

The scalar product of $\vec{A}$ and $\vec{B}$ is zero when $\cos \theta = 0$, i.e. $0 = 90^\circ$

$$\cos 90^\circ = 0 \text{ Put in (i)}$$

$$(i) \quad \vec{A} \cdot \vec{B} = AB(0)$$

$$\vec{A} \cdot \vec{B} = 0$$

Scalar product is zero although $\vec{A}$ and $\vec{B}$ are nonzero.

Zero Vector Product:

The cross product of two nonzero vectors $\vec{A}$ and $\vec{B}$ is zero if $\vec{A}$ and $\vec{B}$ either parallel $||$ or anti parallel $\uparrow \downarrow$ i.e. when angle " θ " between them is " 0° " or " 180° ".

Mathematically:

As the vector product of $\vec{A}$ and $\vec{B}$ is defined as:
 $\vec{A} \times \vec{B} = AB \sin \theta$ --- (ii)

- A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$
- " θ " is the angle between $\vec{A}$ and $\vec{B}$

If angle between them is " 0° " or " 180° " then

$$\sin 0^\circ = 0$$

$$\& \quad \sin 180^\circ = 0 \quad \text{Put in (ii)}$$

$$\Rightarrow \vec{A} \times \vec{B} = AB(0)$$

$$\Rightarrow \vec{A} \times \vec{B} = 0$$

Vector product is zero although $\vec{A}$ and $\vec{B}$ are nonzero.

Conclusion:

For the nonzero vectors $\vec{A}$ and $\vec{B}$ scalar product is zero if $\theta = 90^\circ$ & vector product is zero if $\theta = 0^\circ$ or $\theta = 180^\circ$.

Q.6: Suppose you are given a known nonzero vector $\vec{A}$. The scalar product of $\vec{A}$ with an unknown vector $\vec{B}$ is zero. Likewise, the vector product of $\vec{A}$ with $\vec{B}$ is zero. What can you conclude about $\vec{B}$?

Ans. We are given a nonzero vector $\vec{A}$, if the scalar product of $\vec{A}$ with the unknown vector $\vec{B}$ is zero i.e. $\vec{A} \cdot \vec{B} = 0$ then there are two possibilities.

- Either $\vec{B}$ is zero vector. OR
- Angle between $\vec{A}$ and $\vec{B}$ is 90° i.e. $\theta = 90^\circ$.

Condition of the Question:

But according to the condition of the question the vector product of $\vec{A}$ and $\vec{B}$ is also zero i.e. $\vec{A} \times \vec{B} = 0$

- Since either $A = 0$ or $B = 0$ OR
- Angle between $\vec{A}$ and $\vec{B}$ is (0°) or (180°)
- But the angle between $\vec{A}$ and $\vec{B}$ can either be 90° or $[0^\circ \text{ or } 180^\circ]$
- Two different angles are not possible.
- Thus the only possibility is that $\vec{B} = 0$

Conclusion:

Thus it is concluded that the unknown vector $\vec{B}$ is zero i.e. $\vec{B} = 0$.

Q.7: Why a particle experiencing only one force cannot be in equilibrium?

Ans: Reason:

A body experiencing only one force cannot be in equilibrium because it violates the first condition of equilibrium.

Condition of Equilibrium:

For a body to be in equilibrium, it must obey both the conditions of equilibrium. According to the first condition of equilibrium the sum of all forces acting on a body must be zero i.e. the net force must be zero.

$$\sum \vec{F} = 0$$

$$\vec{F}_{net} = 0$$

Violation of the 1st Condition:

If a body experiences only one force in a given direction the net force acting on the body is not zero, because there is no other force which can cancel the effect of this force i.e. $\vec{F}_{net} \neq 0$

Conclusion:

Since the net force $\vec{F}_{net}$ acting on "a body experiencing only one force is not zero. Therefore it cannot be in equilibrium.

Q. 8: To open a door that has the handle on the right and the hinges on the left a torque must be applied. Is the torque clockwise or counterclockwise when viewed from above? Does your answer depend on whether the door opens towards or away from you?

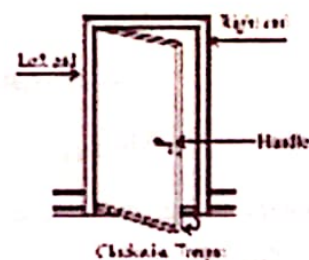
Ans. Torque Applied on the Door:

According to the conditions of the question:

- Handle is on the right end of the door.
- The hinges are on the left side when viewed from the above. The direction of torque applied depends on whether the door opens towards us or opens away from us.

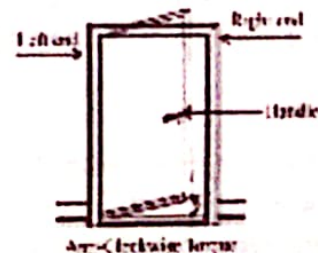
If Door Opens Towards Us:

If the door opens towards us the torque applied is clockwise as is shown in the figure.



Door Opens Away From Us:

If the door opens away from us the torque applied on the door is anti-clockwise as obvious from the figure.



Conclusion:

As viewed from above the

- Torque applied is clockwise if the door opens towards us.

- Torque applied is negative if the door opens away from us.

Q. 9: Explain the warning "Never use a large wrench to tighten a small bolt".

Ans: The warning, "Never use a large wrench to tighten a small bolt" means that if the wrench's opening doesn't exactly fit the size of the bolt it may damage the corners of the bolt or slip, or even break the bolt. If the wrench slips it cannot exert torque on the bolt. Due to these reasons a large wrench should never be used to tighten a small bolt.

Q. 10: A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

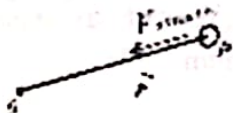
Ans. Torque Due to Central Force:

No, "a central force that is always directed towards the same point" cannot give rise to a torque about that point.

Explanation:

Consider a body at point "P" at some distance from the origin "O".

Figuratively:



$\vec{r}$ Is directed from P to O, $\vec{F}$ is also directed from P to O.

- $\vec{r}$ Is the displacement vector directed from point "P" to "O"
- $\vec{F}$ is the central force acting on the body at point "P" and is always directed towards the origin (point "O")
- It is clear from the figure that $\vec{F}$ and $\vec{r}$ are parallel to each other i.e. angle between them is $\theta = 0^\circ$ so torque is zero.

Mathematically:

Torque is defined as:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin \theta \text{ --- (i) Since } \theta = 0^\circ \text{ and } \sin(0^\circ) = 0$$

$$\text{Thus (i) } \Rightarrow rF(0) \text{ [as angle between } \vec{F} \text{ and } \vec{r} \text{ is zero]} \tau = 0$$

Thus the torque due to the central force $\vec{F}_{\text{attractive}}$ is zero.

Conclusion:

A central force that is always directed towards the point (central point) cannot give rise to a torque about that point.

MCQ's

- Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:
 A. 0° B. 60° C. 70° D. 80°

2. The minimum number of vectors of unequal magnitude require to produce a zero resultant is
(a).2 (B).3 (C).4 (D).5
 3. If the resultant of two vectors, each of magnitude A, is then the angle between the two vectors will be
(A).30° (B).45° (C).60° (D).120°
 4. The magnitude of vector $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$ is
(A).9 (B).5 (C).3 (D).1
 5. When $F_x = 3 \text{ N}$ and $F_y = 5 \text{ N}$ then $F =$
(A).6 N (B).4 N (C).2 N (D).0 N
 6. A meter stick is supported by a knife-edge at the 50 cm mark. Arif hangs masses of 0.40 kg and 0.60 kg from the 20 cm and 80 cm marks respectively. Where should Arif hang a third mass of 0.30 kg to keep the stick balanced?
(A).20 cm (B).70 cm (C).30 cm (D).25 cm
 7. If $A_x = 1.5 \text{ cm}$, $A_y = -1.0 \text{ cm}$, into which quadrant does the vector A point?
(A).I (B).II (C).III (D).IV
 8. $\vec{A} \cdot (\vec{A} \times \vec{B}) = ?$
(A).0 (B).1 (C).AB (D).A²B
 9. Two forces of magnitude 20 N and 50 N act simultaneously on a body. Which one of the following forces cannot be a resultant of these vectors?
(A).20 N (B).30 N (C).40 N (D).70 N
 10. If the dot product of two non-zero vectors A and B is zero, then the magnitude of their cross product is
(A).0 (B).1 (C).AB (D).-AB
 11. The sum of magnitudes of two forces is 16 N. if the resultant force is 8 N and its direction is perpendicular to the minimum force, then the forces are
(A).6 N and 10 N (B).8 N and 8 N (C).4 N and 12 N (D).2 N and 14 N
 12. Find the mass of the uneven rod shown in the figure, if
(A).100 g (B).150 g (C).80 g (D).5 g
- Hint:**
- $$\sum \tau_{\text{clockwise}} = \sum \tau_{\text{anti-clockwise}} \Rightarrow 20 \text{ cm} \times 80 \text{ g} \times 9.81 \text{ cm/s}^2$$
- $$= 16 \text{ cm} \times m \times 9.81 \text{ cm/s}^2 \Rightarrow m = 100 \text{ g}$$
13. The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces F are applied on the rod, as shown in the figure. Which diagram shows the rod in equilibrium?
(A). 30° (B). 45° (C). 60° (D). 90°
 14. For which angle the equation $\vec{A} \cdot \vec{B} = A \times B$ is correct?
(A).30° (B).45° (C).60° (D).90°
 15. What is the net torque on wheel of radius 2 m as shown
(A).10 Nm anti-clockwise (B).5 Nm anti-clockwise
(C).10 Nm clockwise (D).5 Nm clockwise

Solution of MCQs

1. **Sol:** As,

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 80^\circ$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| (0.173)$$

$$\vec{A} \cdot \vec{B} = 0.173 |\vec{A}| |\vec{B}| \text{-----} (*_1)$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 180^\circ$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| (-0.173)$$

$$\vec{A} \cdot \vec{B} = -0.173 |\vec{A}| |\vec{B}| \text{-----} (*_2)$$

From eq $(*_1)$ & $(*_2)$ it is clear that by increasing the angle by 20° the scalar product only changes the sign rather than magnitude.

Hence

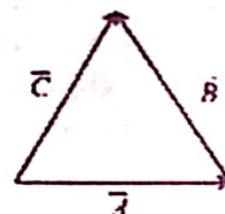
Ans: D. 80°

2. **Sol:**

Ans: B. 3

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

$$0 = \vec{A} + \vec{B} + \vec{C}$$



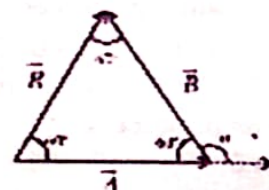
3. **Sol:** In equilateral triangle all the three sides and angles are equal.

In given figure the angle between $\vec{A}$ and $\vec{B}$ is given by

$$\theta = 180 - 60$$

$$\theta = 120^\circ$$

Ans: D. 120°



4. **Sol:** $\vec{A} = 2\hat{i} + 1\hat{j} + 2\hat{k}$

$$|\vec{A}| = \sqrt{(2)^2 + (1)^2 + (2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

Ans: B. 3

5. **Sol:** By Pythagoras theorem

$$F^2 = F_x^2 + F_y^2$$

$$F_y^2 = F^2 - F_x^2$$

$$F_y = \sqrt{F^2 - F_x^2}$$

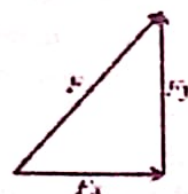
$$F_y = \sqrt{(5)^2 - (3)^2}$$

$$F_y = \sqrt{25 - 9}$$

$$F_y = \sqrt{16}$$

$$F_y = 4N$$

Ans. B. 4N



6. **Sol:** **Ans. A. 30 cm**

7. **Sol:** $\tan \theta = \frac{\text{Perp}}{\text{base}} = \frac{A_y}{A_x}$

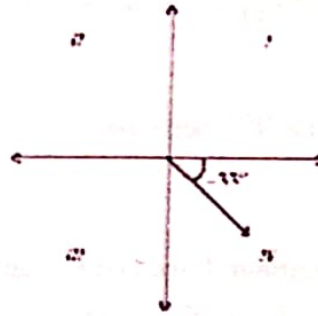
$$\tan \theta = \frac{-1.0}{1.5}$$

$$\tan \theta = -0.66$$

$$\theta = \tan^{-1}(-0.66)$$

$$\theta = -33^\circ$$

Which is the angle from +ve x-axis in clockwise direction. Ans **D. IV**



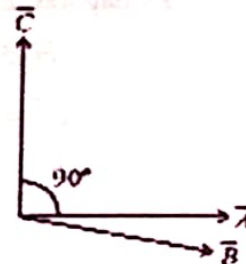
8. **Sol:** Cross product gives a vector quantity whose direction is perpendicular to both $\vec{A}$ and $\vec{B}$.

Let $\vec{A} \times \vec{B} = \vec{C}$

So the angle between $\vec{A}$ and $\vec{C}$ is 90° .

$$\begin{aligned} \vec{A} \cdot \vec{A} \times \vec{B} &= \vec{A} \cdot \vec{C} = |\vec{A}| |\vec{C}| \cos \theta \\ &= |\vec{A}| |\vec{C}| \cos 90^\circ \\ &= |\vec{A}| |\vec{C}| (0) \\ &= 0 \end{aligned}$$

Ans. A.0



9. **Sol:**

Anticlockwise torque = clockwise torque

$$r_1 + r_2 = r_3$$

$$r_1 F_1 + r_2 F_2 = r_3 F_3$$

$$r_1 m_1 g + r_2 m_2 g = r_3 m_3 g$$

$$(r_1 m_1 + r_2 m_2) g = r_3 m_3 g$$

$$r_1 m_1 + r_2 m_2 = r_3 m_3$$

Putting values

$$(30)(0.40) + r_2 m_2 = (30)(0.60)$$

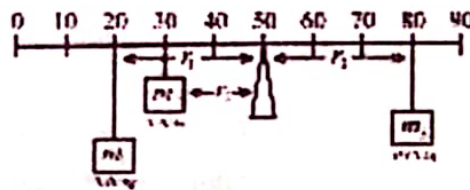
$$(30)(0.40) + r_2 (0.30) = (30)(0.60)$$

$$12 + r_2 (0.30) = 18$$

$$r_2 (0.30) = 18 - 12 = 6$$

$$r_2 = \frac{6}{0.30} = 20$$

Ans. A. 20cm



10. **Sol:** $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$

If $\theta = 90^\circ$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 90^\circ$$

Now $\vec{A} \times \vec{B} = AB \sin \theta$

$$\vec{A} \times \vec{B} = AB \sin 90^\circ$$

$$\vec{A} \times \vec{B} = AB(1)$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\vec{A} \times \vec{B} = AB \sin \theta$$

$$\vec{A} \cdot \vec{B} = 0$$

$\Rightarrow \theta$ will be 90° between

$\vec{A}$ and $\vec{B}$

Hence

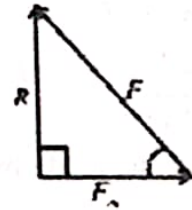
Ans. C. AB

11. Let F_m is the minimum force and F is another force.

Given Data: $F + F_m = 16$ --- (*)

Also given that $R = 8$

Also given that F_m is perpendicular to R as shown in figure by Pythagoras theorem.



$$(\text{Hyp})^2 = (\text{Perp})^2 + (\text{Base})^2$$

$$F^2 = R^2 + F_m^2$$

$$F^2 - F_m^2 = R^2$$

$$F^2 - F_m^2 = (8)^2$$

Put value $R = 8$

$$F^2 - F_m^2 = 64$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(F + F_m)(F - F_m) = 64$$
 --- (*)

Putting eq (*) in eq (*) we get

$$16(F - F_m) = 64$$

$$F - F_m = \frac{64}{16}$$

$$F - F_m = 4$$
 --- (*)

Adding eq (*) and eq (*)

$$(*) \Rightarrow F + F_m = 16$$

$$(*) \Rightarrow F - F_m = 4$$

$$2F = 20$$

$$F = 10N$$

Put in eq (*)

$$F + F_m = 16$$

$$10 + F_m = 16$$

$$F_m = 16 - 10$$

$$F_m = 6N$$

Ans. A. 6N and 10N

12. **Given That:** $m_2 = 80g$

$$r_2 = 20cm$$

$$r_1 = 30 - 14 = 16m$$

Required: $m_1 = ?$

Since 2nd condition of equilibrium is given by

Anticlockwise torque = clockwise torque

$$\tau_1 = \tau_2$$

$$r_1 F_1 = r_2 F_2$$

$$r_1 m_1 g = r_2 m_2 g$$

$$r_1 m_1 = r_2 m_2$$

Putting values

$$16m_1 = (20)(80)$$

$$m_1 = \frac{1600}{16}$$

$$m_1 = 100g$$

Ans. A. 100g

13. **Sol:** Option (C) satisfy the 2nd condition of equilibrium therefore the rod will be in static equilibrium.

Ans = C

14. **Given That:**

$$|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$$

$$AB \cos \theta = AB \sin \theta$$

$$\cos \theta = \sin \theta$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1) = 45^\circ$$

$$\theta = 45^\circ$$

Ans. B. 45°

15. **Given That:**

$$r = 2m$$

$$F_1 = 10N$$

$$F_2 = 5N$$

$$\theta = 90^\circ$$

$$\tau_1 = rF_1 \sin \theta$$

$$\tau_1 = (2)(10) \sin 90^\circ$$

$$\tau_1 = 20NM \text{ Clockwise}$$

$$\tau_2 = rF_2 \sin 90^\circ$$

$$\tau_2 = (2)(5)(1)$$

$$\tau_2 = 10 \text{ Nm} \quad \text{Anticlockwise}$$

$$\tau_N = t_1 - t_2$$

$$\tau_N = 20 - 10$$

$$\tau_N = 10 \text{ Nm}$$

Ans. C

Assignments

Assignment 2.1: An airplane is moving at 120 m/s at an angle of 10° with x-axis, through a 30 m/s cross wind, blowing at angle of 260° with x-axis. Determine the resultant velocity of the airplane.

Given Data:

$$v_1 = 120 \text{ m/s}$$

$$v_2 = 30 \text{ m/s}$$

$$\theta_1 = 10^\circ$$

$$\theta_2 = 260^\circ$$

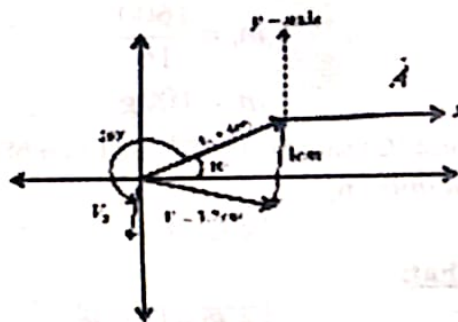
Scale:

$$30 \text{ m/s} = 1 \text{ cm}$$

So

$$\overline{v_1} = 4 \text{ cm}$$

$$\overline{v_2} = 1 \text{ cm}$$



Solution:

Draw vectors according to scale with given directions add according to head-to-tail rule.

Measure length of resultant vector $\overline{v_R}$ as measured in figure.

$$\overline{v_R} = 3.7 \text{ cm}$$

$$\text{But } 1 \text{ cm} = 30 \text{ m/s}$$

So

$$\overline{v_R} = 3.7 \times 30$$

$$v_R = 112.8 \text{ m/s}$$

$$v_R = 113 \text{ m/s}$$

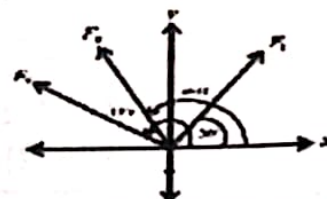
$$\theta_R = 355^\circ$$

Assignment 2.2: A Force $F_1 = 20 \text{ N}$ making an angle $\theta_1 = 30^\circ$ with positive x-axis and Force $F_2 = 30 \text{ N}$ making an angle $\theta_2 = 150^\circ$ with positive x-axis, acts at a point, calculate the resultant force.

Given Data:

$$F_1 = 20 \text{ N}$$

$$\theta_1 = 30^\circ$$



$$F_2 = 30N$$

$$\theta_2 = 150^\circ$$

Required Data:

$$\text{Resultant force} = \overline{F}_R = ?$$

Solution:

By rectangular components

$$\overline{F}_R = \overline{F}_{Rx} + \overline{F}_{Ry}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \text{ ----- (1)}$$

By addition of vectors by rectangular components of each vector.

$$F_{Rx} = F_{1x} + F_{2x} \text{ ----- (2)}$$

$$F_{Ry} = F_{1y} + F_{2y} \text{ ----- (3)}$$

For vector F_1

$$F_{1x} = F_1 \cos \theta_1$$

$$F_{1x} = 20 \cos 30^\circ$$

$$F_{1x} = 17.32^\circ N$$

And

$$F_{1y} = F_1 \sin \theta_1$$

$$F_{1y} = 20 \sin 30^\circ$$

$$F_{1y} = 10^\circ N$$

For vector F_2

$$F_{2x} = F_2 \cos \theta_2$$

$$F_{2x} = 30 \cos 150^\circ$$

$$F_{2x} = -26^\circ N$$

And

$$F_{2y} = F_2 \sin \theta_2$$

$$F_{2y} = 30 \sin 150^\circ$$

$$F_{2y} = 15N$$

Putting values of F_{1x} & F_{2x} in eq (2) we get

$$F_{Rx} = 17.32 + (-26)$$

$$F_{Rx} = 17.32 - 26$$

$$F_{Rx} = -8.7N \text{ ----- (4)}$$

Putting values of F_{1y} & F_{2y} in eq (3) we get

$$F_{Ry} = 10 + 15$$

$$F_{Ry} = 25N \text{ ----- (5)}$$

Putting values of F_{Rx} & F_{Ry} from eq (4) & eq (5) in eq (1), we get

$$F_R = \sqrt{(-8.7)^2 + (25)^2}$$

$$F_R = \sqrt{75.69 + 625} = \sqrt{700.7}$$

$$F_R = 26.46N \quad (\text{Ans})$$

Now to determine angle, we use

$$\theta = \tan^{-1} \left(\frac{F_{Ry}}{F_{Rx}} \right)$$

$$\theta = \tan^{-1} \left(\frac{25.9}{-8.7} \right) = \tan^{-1} (-2.87)$$

$$\theta = -70.81$$

As F_{Rx} is negative and F_{Ry} is positive therefore the resultant lies in 2nd quadrant where.

$$\theta_R = \theta + \pi$$

$$\theta_R = (-70.81) + 180^\circ$$

$$\theta_R = 109.11^\circ \text{ (Ans)}$$

Assignment 2.3: Two forces of 20 N and 10 N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system.

Given Data:

$$\text{Force } F_1 = 20\text{ N}$$

$$\text{Force } F_2 = 10\text{ N}$$

$$\text{Angle between } F_1 \text{ \& } F_2 = \theta_2 = 120^\circ$$

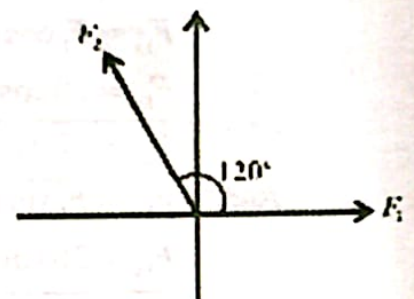


Figure (a)

Required Data:

- Force replace the given force $= R = ?$
- Force which balance the given forces $F = ?$

Solution:

- Let $F_1 = 20\text{ N}$ acting along $+x$ - direction i.e. $\theta_1 = 0$ and $F_2 = 10\text{ N}$ making 120° with $+x$ - direction i.e. $\theta_2 = 120^\circ$.

To find their resultant, we first calculate their x and y components as follows:

$$F_{1x} = F_1 \cos \theta_1 = 20 \cos 0 = 20(1) = 20\text{ N}$$

$$F_{1y} = F_1 \sin \theta_1 = 20 \sin 0 = 20(0) = 0\text{ N}$$

$$F_{2y} = F_2 \sin \theta_2 = 10 \sin 120^\circ = 10(-0.5) = -5\text{ N}$$

$$\text{Thus } R_x = F_{1x} + F_{2x} = 20 + (-5) = 15\text{ N}$$

$$R_x = 15\text{ N}$$

$$\text{And } R_y = F_{1y} + F_{2y} = 0 + 8.66 = 8.66\text{ N}$$

$$R_y = 8.66\text{ N}$$

Hence the magnitude of the resultant is:

$$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{(15)^2 + (8.66)^2}$$

$$|R| = 17.32\text{ N} \quad (\text{Ans})$$

As R_x and R_y both are positive therefore the resultant will lie in 1st quadrant and its direction will be:

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

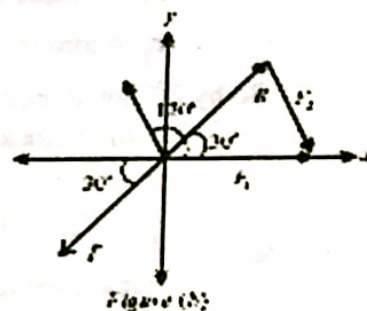
$$\theta = \tan^{-1} \left(\frac{8.66}{15} \right) = \tan^{-1} (0.577) = 30^\circ$$

$$\boxed{\theta = 30^\circ} \quad (\text{Ans})$$

Solution:

- b) The force that will balance the given forces has the same magnitude as their resultant $\bar{R}$ has, but is directed opposite to $\bar{R}$. This is shown in the figure (b).

From figure (b) it is clear that the direction of $\bar{F}$ is $180^\circ + 30^\circ = 210^\circ$ with $+x$ direction in anticlockwise sense.



And $\boxed{\bar{F} = \bar{R} = 17.32N}$

Assignment 2.4: Show That $(\bar{A} \cdot \bar{B})^2 + (\bar{A} \times \bar{B})^2 = A^2 B^2$

Solution: Talking L.H.S

$$\begin{aligned} \Rightarrow (\bar{A} \cdot \bar{B})^2 + (\bar{A} \times \bar{B})^2 &= (AB \cos \theta)^2 + (AB \sin \theta)^2 \\ &= A^2 B^2 \cos^2 \theta + A^2 B^2 \sin^2 \theta \\ &= A^2 B^2 (\cos^2 \theta + \sin^2 \theta) \\ &= A^2 B^2 (1) \\ &= A^2 B^2 \\ &= R.H.S \end{aligned}$$

NOTE
 $\cos^2 \theta + \sin^2 \theta = 1$

Hence L.H.S = R.H.S

Therefore $(\bar{A} \cdot \bar{B})^2 + (\bar{A} \times \bar{B})^2 = A^2 B^2$

Which is required proof.

Assignment 2.5: A uniform plank of 200 N and length 6 m is supported by a rope as shown in the figure. If the breaking tension in the rope is 400 N, how far can a boy of weight 400 N walk towards the support.

Given Data:

Weight of plank $= F_p = 200N$

Length of plank $= l_T = 6m$

Length of center of mass of plank $= l_p = 3m$

Breaking tension in the rope = $F_T = 400N$

Weight of the boy/man = $F_M = 400N$

Required Data:

How far the boy can walk towards support = $x = ?$

Solution:

There will be three torques exerted anticlockwise and two clockwise direction.

Let τ_1 = torque exerted by the bot

τ_2 = torque exerted by plank

τ_3 = torque exerted by rope

So by 2nd law of equilibrium

Sum of clockwise torque = sum of anticlockwise torque

$$\tau_1 + \tau_2 = \tau_3$$

$$F_M x + F_P l_P = F_T l_T$$

Putting values we get

$$400x + 200(3) = 400(6)$$

$$400x + 600 = 2400$$

$$400x = 2400 - 600$$

$$4x = 18$$

$$x = \frac{18}{4}$$

$$x = 4.5m \quad (\text{Ans})$$

Numerical Questions

Numerical Problem 1: A person throws a Ball straight up with a speed of 12 m/s. If the bus is moving at 25 m/s, what is the velocity of the ball to an observer on ground ?

Given Data:

Upward velocity of the ball = $v_b = 12m/sec$

Velocity of the bus = $v_B = 25m/sec$

Required Data:

Velocity of the ball to an observer = $v_R = ?$

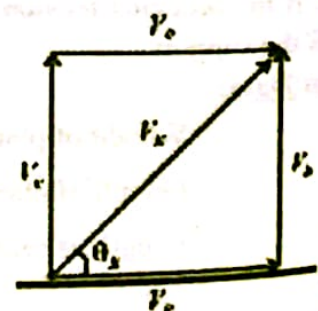
Solution:

From the figure by Pythagoras theorem

$$(Hyp)^2 = (Base)^2 + (Perp)^2$$

$$v_R^2 = v_B^2 + v_b^2$$

Putting values



$$v_R^2 = (25)^2 + (12)^2$$

$$v_R^2 = 625 + 144$$

$$v_R^2 = 769$$

$$v_R = \sqrt{769}$$

$$v_R = 27.7$$

$$v_R = 27 \text{ m/sec}$$

For angle/direction :

As $\tan \theta = \frac{\text{Perp}}{\text{Base}}$

$$\tan \theta = \frac{v_b}{v_B} = \frac{12}{25}$$

Result: $\theta = \tan^{-1}(0.5) = 26^\circ \Rightarrow \theta_R = 26^\circ \text{ (Ans)}$

Numerical Problem 2: A football leaves the foot of a punter at an angle of 54° (positive x-direction) at a speed of 21 m/s. Determine the horizontal and vertical components of the velocity.

Given Data:

Speed of the football $= v = 21 \text{ m/sec}$

Angle $= \theta = 54^\circ$

Required Data:

Horizontal component of velocity $= v_x = ?$

Vertical component of velocity $= v_y = ?$

Solution:

As we know that

$$v_x = v \cos \theta$$

Putting values

$$v_x = (21) \cos(54)$$

$$v_x = (21)(0.59)$$

$$v_x = 12 \text{ m/sec} \text{ ----- (Ans)}$$

We also know that

$$v_y = v \sin \theta$$

Putting values

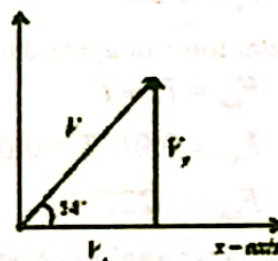
$$v_y = (21) \sin(54)$$

$$v_y = (21)(0.80)$$

$$v_y = 16.9$$

(Rounding off data)

$$v_y = 17 \text{ m/s} \text{ ----- (Ans)}$$



Numerical Problem 3: A 1.84-kg school bag hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50^\circ$. Find the tension T in the clothesline.

Given Data:

Mass of a bag $= m = 1.84 \text{ kg}$

Angle $= \theta = 3.50^\circ$

Required Data:

Tension in clothesline $= T = ?$

Solution:

Resolve tension "T" into its rectangular components.

- Horizontal components of tension will cancel each other because they are opposite in direction and same in magnitude.
- Vertical components of tension "T" will contribute because they are in same direction.

To find the vertical components of tension "T"

As we know that

$$T_y = T \sin \theta$$

$$T_y = T \sin(3.50^\circ)$$

$$T_y = 0.060T$$

Now total force in upward direction is

$$F_{up} = T_y + T_y$$

$$F_{up} = 0.061T + 0.061T$$

$$F_{up} = 1.22T$$

Now the downward force on the bag is given by

$$F_{down} = W = mg \quad (\text{Putting values})$$

$$F_{down} = (1.84)(9.8)$$

$$F_{down} = 18.032 \text{ N}$$

Since the bag is in equilibrium so by 1st condition of equilibrium

$$F_{up} = F_{down}$$

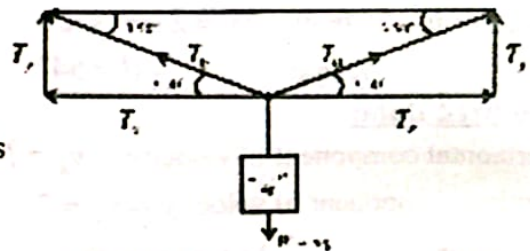
Putting values

$$1.22T = 18.032$$

$$T = \frac{18.032}{1.22}$$

Result:

$$T = 148 \text{ N}$$



Problem 4: Find the magnitude and direction of vectors represented by the following pair of components

(a) $\vec{A}_x = -2.3\text{cm}$, $\vec{A}_y = 4.1\text{cm}$ (b) $A_x = 3.9\text{m}$, $A_y = -1.8\text{m}$

a) Given Data:

$$\vec{A}_x = -2.3\text{cm}$$

$$\vec{A}_y = 4.1\text{cm}$$

Required Data:

$$\vec{A} = ?$$

$$\theta_A = ?$$

Solution:

As we know that

$$A^2 = A_x^2 + A_y^2$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{(-2.3)^2 + (4.1)^2}$$

$$A = \sqrt{5.29 + 16.81}$$

$$A = \sqrt{22.1}$$

$$\vec{A} = 4.7\text{cm} \quad (\text{Ans})$$

Now to find " θ "

$$\text{We know that } \tan \theta = \frac{\text{Perp}}{\text{Base}}$$

$$\tan \theta = \frac{A_y}{A_x}$$

$$\tan \theta = \frac{4.1}{-2.3}$$

$$\theta = \tan^{-1} \left(\frac{-4.1}{2.3} \right)$$

$$\theta = -60.7^\circ = -61^\circ$$

As A_x is $-ve$ and A_y is $+ve$ therefore the resultant lies in 2nd quadrant

Where $\theta_A = \pi + \theta$

$$\theta_A = 180 - 60.7$$

$$\theta_A = 119.3^\circ$$

b) Given Data:

$$A_x = 3.9\text{m}$$

$$A_y = -1.8\text{m}$$

Required Data:

$$A = ?$$

$$\theta_A = ?$$

Solution:

To find "A"

We know that

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{(3.9)^2 + (-1.8)^2}$$

$$A = \sqrt{15.2 + 3.24}$$

$$A = \sqrt{18.44}$$

$$\boxed{A = 4.3}$$

Result:Now to find " θ "

$$\text{We know that } \tan \theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

$$\theta = \tan^{-1} \left(\frac{-1.8}{3.9} \right)$$

$$\theta = -24.8$$

As A_x is +ve and A_y is -ve therefore the resultant lies in 4th quadrant

$$\text{Where } \theta_A = 360^\circ + \theta = 360^\circ - 24.8^\circ = 335.2^\circ$$

Result:

$$\boxed{\theta_A = 335.2^\circ}$$

Numerical Problem 5: Vector $\vec{F}$ having magnitude 5.5 N makes 10° with x-axis and vector with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products?

Given Data:

$$\text{Magnitude of force} = \vec{F} = 5.5 \text{ N}$$

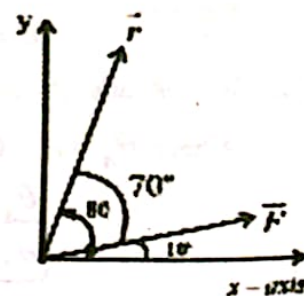
$$\text{Magnitude of vector} = \vec{r} = 4.3 \text{ m}$$

$$\text{Angle b/w force and vector } \theta = 80^\circ - 10^\circ = 70^\circ$$

Required Data:

$$\text{a) } \vec{F} \times \vec{r} = ?$$

$$\text{b) } \vec{F} \cdot \vec{r} = ?$$

**Solution:**To find $\vec{F} \times \vec{r} = ?$

$$\vec{F} \times \vec{r} = Fr \sin \theta$$

Putting values

$$\vec{F} \times \vec{r} = (5.5)(4.3) \sin 70^\circ$$

$$\vec{F} \times \vec{r} = (23.65)(0.9)$$

Solution:To find $\vec{F} \cdot \vec{r} = ?$

$$\vec{F} \cdot \vec{r} = Fr \cos \theta$$

Putting values

$$\vec{F} \cdot \vec{r} = (5.5)(4.3) \cos 70^\circ$$

$$\vec{F} \cdot \vec{r} = (23.65)(0.34)$$

$$\vec{F} \times \vec{r} = 22.2 \text{ Nm}$$

$$\vec{F} \cdot \vec{r} = 8.1 \text{ Nm}$$

Problem 6: The magnitude of dot and cross product of two vectors $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

Given Data:

$$\vec{A} \cdot \vec{B} = AB \cos \theta = 6\sqrt{3}$$

$$\vec{A} \times \vec{B} = AB \sin \theta = 6$$

Required Data

Angle b/w two vectors = $\theta = ?$

Solution:

As we are given that

$$AB \cos \theta = 6\sqrt{3} \text{ ----- (i)}$$

$$AB \sin \theta = 6 \text{ ----- (ii)}$$

Dividing eq (ii) by eq (i) we get

$$\frac{AB \sin \theta}{AB \cos \theta} = \frac{6}{6\sqrt{3}}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\theta = 30^\circ \text{ ----- (Ans)}$$

Problem 7: A uniform rod 1m long with weight 6 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance?

Given Data:

Length of the rod = $l = 1 \text{ m}$

Weight = $W_1 = 10 \text{ N}$

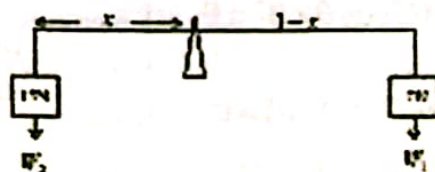
Weight = $W_2 = 15 \text{ N}$

Required Data:

Position of the point of balance = $x = ?$

Solution:

Let "x" be the point of balance from the left end of the rod.



Since the rod is in equilibrium so by 2nd condition of equilibrium

Anticlockwise torque = Clockwise torque

$$xW_2 = (1-x)W_1$$

Putting values

$$x(15) = (1-x)(10)$$

$$15x = 10 - 10x$$

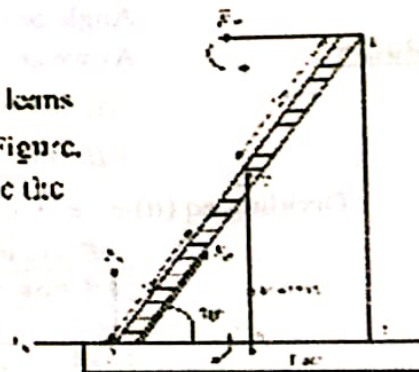
$$15x + 10x = 10$$

$$25x = 10$$

$$x = \frac{10}{25}$$

$$x = 0.41m$$

Problem 8: A 4.0-m-long uniform ladder with weight of 120 N leans against a wall making 70° above a cement floor as shown in Figure. Assuming the wall is frictionless, but the floor is not, Determine the forces exerted on the ladder by the floor and by the wall.



Given Data:

Weight of ladder = $W = 120N$

$$\theta = 70^\circ$$

Length of ladder = $L = 4m$

Required Data:

Reaction of wall $F_W = ?$

Reaction of ground $F_G = ?$

Solution: According to 1st condition of equilibrium

$$\sum F_x = 0$$

And

$$\sum F_y = 0$$

$$F_H - F_W = 0$$

$$F_N - W = 0$$

$$F_H = F_W \text{ --- (i)}$$

$$F_N = W$$

$$F_N = 120N \text{ --- (ii)}$$

Here F_N is Normal force

For F_H is the horizontal force or force of friction

Now according to 2nd condition of equilibrium

$$\sum \tau = 0$$

$$\overline{BD} \times F_W - \overline{AE} \times W = 0$$

$$AB \sin \theta \times F_W - AC \cos \theta \times W = 0$$

Putting values

$$4 \sin 70^\circ F_W - 2 \cos 70^\circ (120) = 0$$

$$4(0.93) F_W - 2(0.34)(120) = 0$$

NOTE

Torque due to F_H and F_N are zero, because their moments arms are zero.

$$BD = AB \sin \theta$$

$$AE = AC \cos \theta$$

$$3.72F_w - 82.08 = 0$$

$$3.72F_w = 82.08$$

$$F_w = \frac{82.08}{3.72}$$

$$F_w = 22.06N$$

This is the reaction force of the wall on the ladder.

Now to find $F_G = ?$

Since $F_N = 20N$

$$F_H = F_w = 22N$$

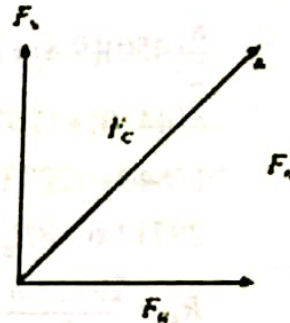
By Pythagoras theorem

$$F_G = \sqrt{F_H^2 + F_N^2}$$

$$F_G = \sqrt{(22)^2 + (120)^2} = \sqrt{484 + 14400} = \sqrt{14884} = 122N$$

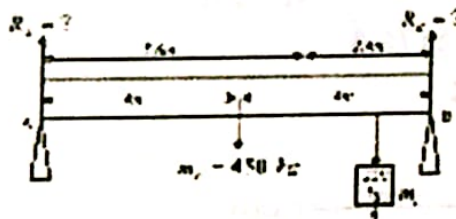
$$F_G = 122N \text{ ----- (Answer)}$$

Which is the reaction force of the ground on the ladder.



Numerical Problem 9: The 450-kg uniform beam supports the load of 220 kg as shown.

Determine the reactions at the supports.



Given Data:

Mass of beam $m_b = 450kg$

Mass of load $m_l = 220kg$

Length of beam $= L = 8m$

Required Data:

$$R_A = ?$$

$$R_B = ?$$

Solution: Let $\tau_b =$ Torque due to beam of mass 450 kg

$\tau_l =$ Torque due to load

$\tau_{RA} =$ Torque due to R_A

$\tau_{RB} =$ Torque due to R_B

Fix Point A:

By 2nd condition of equilibrium

Clockwise torque = anticlockwise torque

$$\tau_b + \tau_l = \tau_{RB}$$

$$\frac{L}{2}m_b g + r_l m_l g = LR_B$$

$$\frac{8}{2}(450)(9.8) + (5.6)(220)(9.8) = 8R_B$$

$$4(4410) + 12073.6 = 8R_B$$

$$17640 + 12073.6 = 8R_B$$

$$29713.6 = 8R_B$$

$$R_B = \frac{29713.6}{8} = 3714.2$$

$$R_B = 3714N \text{ ----- (Ans)}$$

$$\tau = rF$$

$$\tau = rW$$

$$\tau = rmg$$

Now Fix Point B:

By 2nd condition of equilibrium

Sum of clockwise torque = sum of anticlockwise torque

$$\tau_{RA} = \tau_b + \tau_l$$

$$LR_A = r_b m_b g + r_l m_l g$$

$$8R_A = 4(450)(9.8) + (2.4)(220)(9.8)$$

$$8R_A = 17640 + 5174.4$$

$$8R_A = 22814.4$$

$$R_A = 2851.8$$

$$R_A = 2852N \text{ ----- (Ans)}$$

Unit # 3

Forces & Motion

Exercise Long Questions

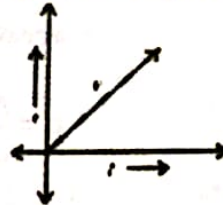
Q1. Explain displacement-time graph and velocity-time graph. In each type give brief detail along with appropriate diagrams for illustration.

Ans. Displacement-Time Graph:

The graph obtained, when the displacement covered by a body is plotted against (versus) time is called displacement-time graph it gives velocity of the body.

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}$$

$$\bar{v} = \frac{\Delta \bar{s}}{\Delta t} = \frac{\bar{s}_f - \bar{s}_i}{t_f - t_i}$$

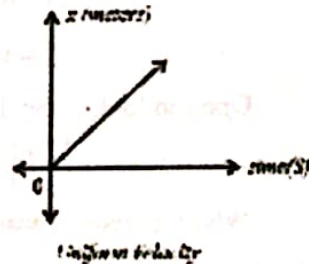


Where $\Delta \bar{s} = \Delta y$, $\Delta x = \Delta t$, time is independent (ازاد) variable and is conventionally taken along x-axis while displacement (dependent) is taken along y-axis.

Uniform Velocity:

If the slope of the line (graph) is a straight line (constant), the velocity of the body is uniform as shown by the graph.

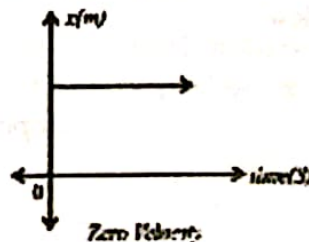
$$\text{Slope} = \frac{\text{Equal displacement}}{\text{Equal time intervals}}$$



Zero Velocity:

If the slope of $(x-t)$ graph is zero, the velocity of the body is zero, as shown below.

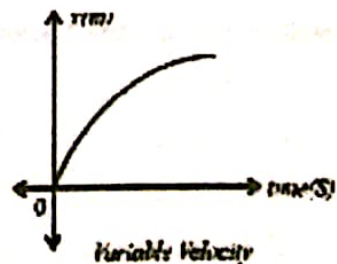
$$\text{slope} = \frac{\Delta x}{\Delta t} = 0$$



Variable Velocity:

If the slope of the displacement-time graph is changing, it gives variable velocity as shown in figure.

$$\text{Slope} = \frac{\text{Unequal displacement}}{\text{time}}$$



As slope is steep at the start of motion so the velocity is high. As slope becomes less steep velocity decreases.

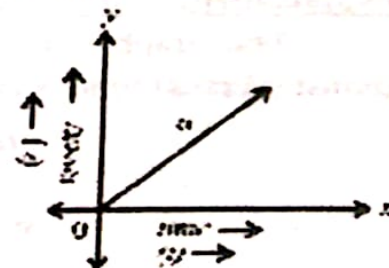
Velocity-Time Graph (v-t):

The graph plotted between velocity and time is called velocity-time graph. The slope of the velocity-time graph gives acceleration as shown below.

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}}$$

$$\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$$



Where $\Delta y = \Delta \bar{v}$, $\Delta x = \Delta t$, time is independent variable, is taken along x-axis, while velocity is taken along y-axis.

Area Under Velocity-Time Graph:

Area under velocity-time graph is equal to distance (displacement) covered by the body. As area of (v-t) graph is given as:

$$A = \text{length} \times \text{width}$$

$$A = v \cdot t \text{-----(1)}$$

One can look at the dimensions of $v \cdot t$ (v multiplied by t) as:

$$[\bar{v}t] = [LT^{-1} \cdot T] = [L]$$

Which gives dimensions of length or distance hence area under (v-t) graph is the distance covered by the body.

Uniform Acceleration:

If the slope of the line (graph) is constant, the acceleration is uniform as given below.

$$\text{slope} = \frac{\text{equal } \Delta \bar{v}}{\text{equal time interval}}$$



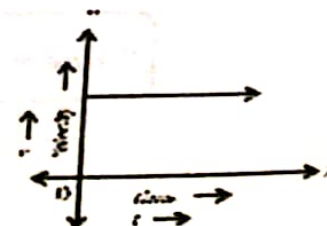
(Uniform or constant acceleration)

Zero Acceleration:

If the slope of the line of (v-t) graph is zero, the acceleration of the body will be zero, as shown below.

$$\text{slope} = 0$$

$$\bar{a} = 0$$

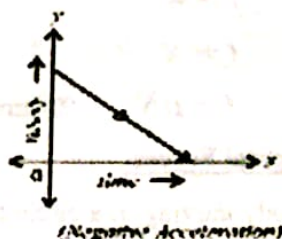


(Zero acceleration)

Negative Acceleration:

If the slope of the ($v-t$) graph decrease, the acceleration of the body will be negative also called deceleration or retardation.

$$\text{slope} = \frac{\text{decreasing } \bar{v}}{\text{time}}$$



Q.2. Apply Newton's laws to explain the motion of objects in a variety of context.

Ans. Applying Newton's Laws to Explain the Motion of Objects in a Variety of Context:

Newton's First Law:

A body at rest will remain at rest and a body in motion will remain in uniform motion in a straight line, unless an external force acts on the body.

Mathematically:

$$\begin{aligned} \text{If } \bar{F} = \bar{0} &\Rightarrow \bar{a} = \bar{0} \\ &\Rightarrow \text{Either } \bar{v} = \bar{0} \text{ (rest)} \\ \text{OR } &\bar{v} = \text{constant} \end{aligned}$$

Applying Newton's First Law:

$$\text{Equation } \sum \bar{F} = 0$$

Is used to solve the problems concerned with equilibrium of bodies.

$$\text{Equation } \bar{F} = 0$$

Is used for the law of conservation of linear momentum, to calculate momentum (quantity of motion) of the objects.

Newton's Second Law of Motion:

When net force acts on an object, it produces acceleration, this acceleration is directly proportional to the magnitude of applied force and inversely proportional to the mass of the body.

Mathematically:

$$\begin{aligned} &\bar{a} \propto \bar{F} \\ &\bar{a} \propto \frac{1}{m} \\ \text{OR } &a \propto \frac{F}{m} \\ \Rightarrow &a = K \frac{F}{m} \\ \Rightarrow &\boxed{F = ma} \end{aligned}$$

$$\boxed{K=1}$$

Equation for Newton's second law.

Applying Newton's Second Law of Motion:

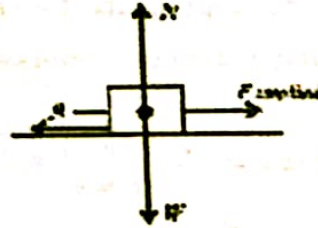
For Friction:

It is used to solve problem concerned with friction of a block placed on a surface as shown.

$$f_s \propto W$$

OR $f_s \propto N$

$$f_s = \mu N \quad \text{Where } N = mg$$



For Centripetal Forces:

A body moving in a circle has centripetal acceleration $a_c = \frac{v^2}{r}$ the centripetal force can be calculated by applying $F = ma$

$$\Rightarrow F_c = m \left(\frac{v^2}{r} \right)$$

$$\Rightarrow F_c = \frac{mv^2}{r}$$



For Miscellaneous Problems:

Newton's second law can be applied to a variety of daily life mechanical problems:

- i) Walking, jumping, collisions, dragging etc.
- ii) Pushing a car
- iii) Kicking a ball
- iv) Speeding up, slowing down of vehicles

Newton's Third Law of Motion:

For every action there is equal but opposite reaction.

Mathematically:

$$\vec{F}_{\text{action}} = -\vec{F}_{\text{reaction}}$$

Action and reaction forces never neutralize each other.

Applying Newton's Third Law of Motion:

Newton's third law of motion is applicable to a variety of systems.

- i) Motion of rocket
- ii) Walking on the ground
- iii) Firing of pistol
- iv) Explosion of bomb etc.
- v) Hitting a cricket ball etc.

Conclusion:

In nearly all mechanical systems, Newton's laws of motion are applied to study and interpret the motion of objects. These laws are the backbone of Newtonian mechanics.

Q3. What is linear momentum? Derive and state Newton's second law in terms of linear momentum.

Ans. Linear Momentum:

The product of mass and linear velocity is called linear momentum.

Mathematically:

$$\vec{p} = m\vec{v}$$

Nature of Momentum:

Momentum is a vector quantity and its direction is that of the velocity of the body.

Unit:

The unit of momentum is kg m/s or N.S.

Dimensions of Momentum:

The dimensions of momentum are $[M L T^{-1}]$

Newton's Second Law and Linear Momentum:

The rate of change of linear momentum is equal to the force ($\vec{F}$) acting on the body.

Derivation:

From Newton's Second law

$$\vec{F} = m\vec{a} \text{ ----- (i)}$$

$$\text{Also } \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \text{ or } \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \text{ ----- (ii)}$$

Put (ii) in (i)

$$\vec{F} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

$$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$\text{OR } \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \text{ ----- (iii)}$$

Hence the rate of change of linear momentum $\left(\frac{\Delta \vec{p}}{\Delta t}\right)$ is equal to the force $\vec{F}$ acting the body.

Equation (iii) is the exact form of Newton's 2nd law of motion. The basic form

$F = ma$ is not applicable in some cases but this form is applicable in almost all cases of motion.

Q4. State and explain law of conservation of linear momentum for an isolated system of bodies.

Ans. Statement of the Law of Conservation of Linear Momentum:

In an isolated system the total momentum of a system of bodies remain constant. OR

For an isolated system the initial momentum of bodies (body) will be equal to final momentum of bodies (body).

Mathematically:

$$\bar{p} = \text{Constant}$$

$$\Rightarrow \Delta \bar{p} = 0$$

$$\text{OR } \bar{p}_f - \bar{p}_i = 0$$

$$\text{OR } \bar{p}_f = \bar{p}_i$$

This implies that the initial momentum is equal to the final momentum.

Conservation of momentum for an isolated system of bodies:-

Consider a two body system of bullet and pistol.

Momentum Before Firing:

Before firing of pistol initial momentum of bullet (m_1) and pistol (m_2) are zero as both are at rest (zero velocities) i.e. $\bar{p}_i = 0$ ----- (i)

Momentum After Firing:

After firing bullet gets velocity $\bar{v}_1$ and pistol velocity $\bar{v}_2$ and move in opposite direction final momentum is:

$$\bar{p}_f = \bar{p}_{\text{bullet}} + \bar{p}_{\text{pistol}}$$

$$\bar{p}_f = m_b \bar{v}_b + m_p \bar{v}_p \text{ ----- (ii)}$$

According to law of conservation of linear momentum:

$$\bar{p}_i = \bar{p}_f$$

$$\text{OR } 0 = m_b \bar{v}_b + m_p \bar{v}_p$$

$$\text{OR } m_b \bar{v}_b = -m_p \bar{v}_p \text{ ----- (iii)}$$

Equation (iii) shows that bullet and pistol gains equal momenta, although different masses. But to make the momentum equal, the bullet gains higher velocity than the pistol.

The negative sign shows that, since the initial momentum before firing was zero, so to make it equal to zero the bullets and pistol after getting equal momenta must move in opposite directions.

NOTE:

The law of conservation of linear momentum can be applied to many body isolated system in the same way.

Q5. Define elastic and inelastic collisions. Give examples in each case, derive mathematical equations for calculating the final velocities of the elastically colliding bodies in one dimension.

Ans. Elastic Collision:

The collision in which kinetic energy linear momentum, and total energy remain conserved (محفوظ) is called elastic collision.

Mathematically:

$$\text{Initial K.E} = \text{Final K.E}$$

$$\text{Initial } \bar{p} = \text{Final } \bar{p}$$

$$\text{Initial } E = \text{Final } E$$

OR

$$K.E_i = K.E_f$$

$$\bar{p}_i = \bar{p}_f$$

$$E = E$$

Examples:

- i) Collision of billiard balls
- ii) Collision of nuclei or atoms

Inelastic Collision:

The collision in which linear momentum ($\bar{p}$) and total energy (E) remain conserved but kinetic energy does not remain conserved.

Mathematically:

$$\bar{p}_i = \bar{p}_f$$

$$E_i = E_f$$

$$\text{But } K.E_i \neq K.E_f$$

K.E is not conserved because a part of it is converted to heat, light or some other form of energy.

Examples:

- i) Collision of wet mud balls
- ii) Collision of bullet with wooden block

Elastic Collision in One Dimension:

The elastic collision in which two bodies, before and after collision move along the same line is called elastic collision in one dimension. Consider two spherical masses m_1 & m_2 move with velocities v_1 and v_2 , the two bodies collide elastically and move after collision with velocities v_1 and v_2 , as shown in figure.

**Conservation of Momentum:**

$$P_i = P_f$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Rearrange and take m common.

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \text{ --- (i)}$$

Conservation of Kinetic Energy:

$$K.E_i = K.E_f$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Rearrange and multiply both sides by 2

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2) \text{ ----- (ii)}$$

Divide equation (ii) by (i)

$$\frac{m_1 (u_1^2 - v_1^2)}{m_1 (u_1 - v_1)} = \frac{m_2 (v_2^2 - u_2^2)}{m_2 (v_2 - u_2)}$$

$$\text{Use } a^2 - b^2 = (a+b)(a-b)$$

$$u_1 + v_1 = u_2 + v_2 \text{ ----- (iii)}$$

OR $u_1 - u_2 = -(v_1 - v_2)$

$$u_{rel} = -v_{rel}$$

u_{rel} = Relative velocity of approach

v_{rel} = Relative velocity of recession

Velocity of Colliding Bodies After Collision:

Use equation (iii)

$$u_1 + v_1 = u_2 + v_2$$

OR $v_2 = u_1 + v_1 - u_2 \text{ ----- (iv)}$

Put equation (iv) in equation (i)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 + v_1 - u_2)$$

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_1 + m_2 v_1 - m_2 u_2$$

Take v_1 as common

$$\Rightarrow m_1 u_1 + m_2 u_2 + m_2 u_2 - m_2 u_1 = v_1 (m_1 + m_2)$$

OR $v_1 = \frac{(m_1 - m_2) u_1}{m_1 + m_2} + \frac{2 m_2 u_2}{m_1 + m_2} \text{ ----- (v)}$

Similarly the final velocity of m_2 i.e. v_2 is given as:

$$v_2 = \frac{(2 m_1) u_1}{m_1 + m_2} - \frac{(m_1 - m_2)}{m_1 + m_2} u_2$$

NOTE:

Collision can occur without physical contact (touch) between bodies, as collision of α -particles with gold nucleus in Rutherford Atomic Model experiment.

If the air resistance and other influencing forces are neglected the motion of projectile is then called ideal projectile motion.

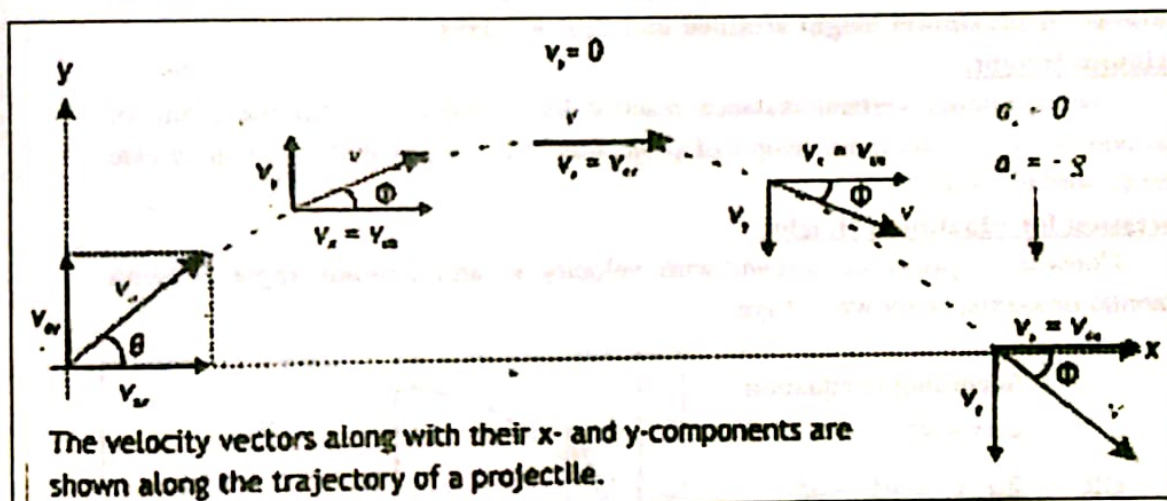
The path of projectile is called trajectory and the study of motion of projectile is called ballistics. Galileo showed that path followed by projectile is parabolic but sometimes it may be linear.

Examples:

- i) Football or cricket ball hit into air
- ii) Shell fired from a cannon
- iii) Stone thrown down a hill

Expression for Instantaneous Velocity of Projectile Motion:

Consider a projectile is thrown with certain velocity v_0 making an angle θ with x-axis. The horizontal component of velocity v_x remains constant, as no force acts along x-axis (ideally) so v_x doesn't change while vertical component of velocity v_y changes at the rate of 9.8 m/s/s (constantly changing) throughout the flight.



For X-Axis:

$$v_{fx} = v_{ix} + at$$

Here $v_{fx} = v_x, v_{ix} = v_0 \cos \theta$

$$a_x = 0, t = t$$

Hence $v_x = v_0 \cos \theta + (0)t$

It shows $v_{ix} = v_{fx} = v_0 \cos \theta$

For Y-Axis:

$$v_{fy} = v_{iy} + a_y t$$

Here $v_{fy} = v_y, v_{iy} = v_0 \sin \theta$

$$a_y = -g, \quad t = t \quad \text{Special convention of "g" i.e.}$$

$$a_y = -g$$

$$\text{Hence } v_y = v_0 \sin \theta - gt$$

Magnitude of Instantaneous Velocity:

$$\text{Use } v = \sqrt{v_x^2 + v_y^2}$$

$$\text{OR } v = \sqrt{(v_0 \cos \theta)^2 + (v_0 \sin \theta - gt)^2}$$

Direction of Instantaneous Velocity:

Use trigonometric relation

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right)$$

$$\text{OR } \theta = \tan^{-1} \left(\frac{v_0 \sin \theta - gt}{v_0 \cos \theta} \right)$$

Q. 7. What is maximum height and time of flight for projectile? Derive mathematical equations for maximum height attained and time of flight.

Ans. Maximum Height:

The maximum vertical distance reached by a projectile from the plane of projection is called maximum height of projectile. It is represented by H. It is also called as summit height.

Expression for Maximum Height:

Consider a projectile thrown with velocity v_0 and making angle θ with horizontal or x-axis, as shown in figure.

According to equation

$$2as = v_f^2 - v_i^2$$

$$\text{OR } 2a_y s_y = v_{fy}^2 - v_{iy}^2$$

$$v_{fy} = 0, \quad v_{iy} = v_0 \sin \theta$$

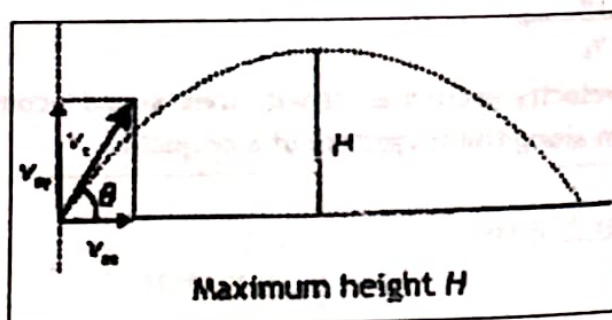
$$a_y = -g, \quad s_y = H$$

Implies that

$$2(-g)H = (0)^2 - (v_0 \sin \theta)^2$$

$$-2gH = -v_0^2 \sin^2 \theta$$

$$\text{OR } H = \frac{v_0^2 \sin^2 \theta}{2g} \quad \text{Required Expression.}$$

**Time of Flight:**

The time taken by a projectile from the point of projection to the point of impact is called time of flight. It is represented by T.

Expression for Time of Flight:

Consider a projectile which is shown with a velocity v_0 making an angle θ with x-axis as shown:

Using

$$s = v_i t + \frac{1}{2} a t^2$$

$$s_y = v_{iy} t + \frac{1}{2} a_y t^2$$

Here

$$v_y = 0, \quad v_{iy} = v_0 \sin \theta$$

$$a_y = -g, \quad t = T$$

Put values

$$0 = v_0 \sin \theta T - \frac{1}{2} g T^2$$

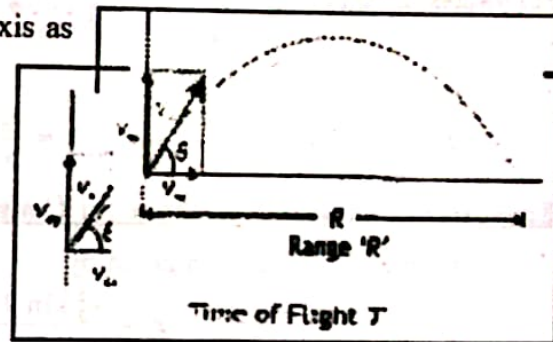
OR

$$\frac{1}{2} g T^2 = v_0 \sin \theta T$$

OR

$$T = \frac{2 v_0 \sin \theta}{g}$$

Required Expression



Time for Summit Height:

Time taken by projectile to reach maximum height is called time for summit height. It is equal to half $\left(\frac{1}{2}\right)$ of the time of flight for a projectile i.e.

$$T' = \frac{1}{2} T$$

$$\text{OR } T' = \frac{1}{2} \left(\frac{2 v_0 \sin \theta}{g} \right)$$

$$\text{OR } T' = \frac{v_0 \sin \theta}{g}$$

- Q8. What is range of projectile? State in which condition the range will be maximum if speed of projection is kept constant in a uniform gravitational field. Also show that there are two projection angles for the same range.

Ans. Range of Projectile:

The horizontal distance from the point of projection to the point of impact covered by projectile is called range of projectile. Its symbol is R.

Expression for Range of Projectile:

Consider a projectile thrown with velocity v_0 making angle θ with x-axis as shown in figure.

Applying relation

$$s = v_i t + \frac{1}{2} a t^2$$

$$s_x = v_{ix} t + \frac{1}{2} a_x t^2 \dots \dots \dots (1)$$

Here

$$v_{ix} = v_0 \cos \theta, \quad v_x = R, \quad a_x = 0$$

$$T = \frac{2 v_0 \sin \theta}{g} \quad (\text{Time of flight})$$

Put values in eq (1)

$$R = v_0 \cos \theta \left(\frac{2v_0 \sin \theta}{g} \right) + \frac{1}{2}(0) \left(\frac{2v_0 \sin \theta}{g} \right)^2$$

$$R = \frac{2v_0 \sin \theta \cos \theta}{g} \quad \text{or} \quad R = \frac{v_0^2 (2 \sin \theta \cos \theta)}{g} \quad R = \frac{v_0 \sin 2\theta}{g}$$

Expression of Maximum Range for Constant Speed v_0 :

Range of projectile is given by

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

If v_0 is constant, as g is also a constant so range depends on $\sin 2\theta$ only. So for maximum range:

$$\sin 2\theta = \max$$

OR $\sin 2\theta_{\max} = 1$ (one is max value of $\sin \theta$)

OR $2\theta_{\max} = \sin^{-1}(1)$ as $\sin^{-1}(1) = 90^\circ$

OR $2\theta_{\max} = 90^\circ$

OR $\theta_{\max} = \frac{90^\circ}{2}$

OR $\theta_{\max} = 45^\circ$

Conclusion:

Maximum range is achieved by a projectile if the angle of projection is 45° i.e. $\theta = 45^\circ$.

Two Projection Angles for the Same Range:

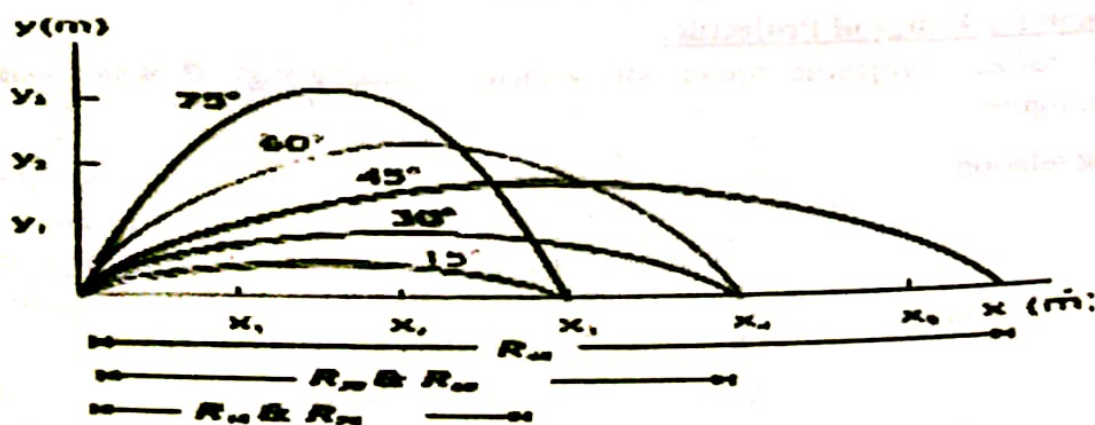
For a given pair of complementary angles i.e. $\theta_1 + \theta_2 = 90^\circ$. The range of projectile will be the same.

Examples:

For $(15^\circ \text{ \& } 75^\circ)$, $(30^\circ \text{ \& } 60^\circ)$ and 45° etc. the range of projectile will be the same.

Figuratively:

This has been shown figuratively as:



Projectile at two complementary angles

Non-exercise Long Non-Exercise long Questions

Q1. Define rest and motion. Rest and motion are relative comment?

Ans. Rest:

The state of a body in which it does not change its position with respect to an observer is said to be in a state of rest.

Example:

- A book lying on a table.
- A chair resting on ground.

Motion:

The state of a body in which a body changes its position with respect to an observer is called motion.

Example:

- Motion of vehicle on road.
- Motion of blades of fan.

Rest and Motion are Relative:

The state of rest or motion of a body depends upon the state of the observer. A body may be at rest with respect to an observer but it may be in motion with respect to another observer and vice versa.

Example:

A person sitting on the chair relative to an observer on earth is at rest but with respect to an observer on the surface of moon he is in motion.

Conclusion:

Thus there is no absolute rest or motion rather rest and motion are relative.

Q2. Define and explain displacement, velocity and acceleration?

Ans. Displacement:

The shortest directed distance between two points is called displacement.

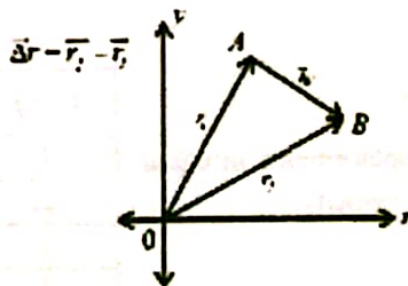
Nature of Displacement: Displacement is a vector quantity.

Unit: The unit of displacement is meter.

Dimensions: The dimensions of displacement are [L].

Formula for Displacement:

Consider the motion of a body between two points A and B, identified by position vectors $\vec{r}_i$ and $\vec{r}_f$ respectively as shown in figure. The displacement $\Delta \vec{r}$ of the body is a straight directed line from point A to point B.



Velocity:

The rate of change of displacement with respect to time is called velocity.

Symbol:

The symbol of velocity is $\vec{v}$.

Mathematically

$$\text{Velocity} = \frac{\Delta s}{\Delta t}$$

$$\vec{v} = \frac{s_f - s_i}{t_f - t_i}$$

Nature:

Velocity is a vector quantity.

Unit of velocity:

The unit of velocity is meter per second $[m/s]$ as

$$\vec{v} = \frac{s_f - s_i}{t_f - t_i}$$

$$\text{Unit of } \vec{v} = \frac{\text{unit of } \vec{s}}{\text{unit of } t}$$

$$\text{Unit of } \vec{v} = \frac{m}{s} \text{ or } \boxed{m/s}$$

Dimensions of Velocity:

The dimensions of velocity are $[LT^{-1}]$ or $\left[\frac{L}{T}\right]$

$$\text{As } \vec{v} = \frac{\Delta s}{\Delta t}$$

$$[\vec{v}] = \left[\frac{\Delta s}{\Delta t}\right]$$

$$[\vec{v}] = \left[\frac{L}{T}\right][LT^{-1}]$$

Average Velocity:

Total displacement covered by body divided by total time is called average velocity.

$$\langle \vec{v} \rangle = \frac{\text{Total displacement}}{\text{Total Time}} \quad \text{OR}$$

$$\langle \vec{v} \rangle = \frac{\vec{s}}{t}$$

Uniform Velocity:

If a body covers equal displacements in equal intervals of time is called uniform velocity.

$$\vec{v} = \frac{\vec{s}}{t}$$

$t(s)$	$\vec{s}(m)$
1	2
2	4
3	6
4	8

Instantaneous (المحتمى) Velocity:

The velocity of a body at a particular instant of time is called instantaneous velocity.

Mathematically:

To get exact or nearly accurate instantaneous velocity the time interval should be taken very small. i.e.

$$\bar{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

When a body moves with uniform velocity its average and instantaneous velocities are equal.

Acceleration:

The rate of change of velocity (speed) with respect to time is called acceleration.

Symbol:

The symbol of acceleration is $\bar{a}$.

Mathematically:

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$$

$$\bar{a} = \frac{\bar{v}_f - \bar{v}_i}{t_f - t_i}$$

Unit:

The unit of acceleration is m / s^2 or ms^{-2} .

As

$$\text{Unit of } \bar{a} = \frac{\text{unit of } \bar{v}}{\text{unit of } t}$$

$$\text{Unit of } \bar{a} = \frac{m/s}{s} = \frac{m}{s} \cdot \frac{1}{s}$$

$$\text{Unit of } \bar{a} = \frac{m}{s} \times \frac{1}{s} = m/s^2 \text{ or } ms^{-2}$$

Nature of Acceleration:

Acceleration is a vector quantity.

Dimensions of Acceleration:

As

$$\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$$

$$[\bar{a}] = \left[\frac{LT^{-1}}{T} \right]$$

$$[\bar{a}] = [LT^{-2}]$$

Positive Acceleration:

If the speed (velocity) of a body increases with time then acceleration is called positive acceleration. i.e. $\overline{v}_f > \overline{v}_i$

$$\overline{a} > 0$$

Negative Acceleration:

If the speed (velocity) of a body decreases with time is called negative acceleration. i.e. $\overline{v}_f < \overline{v}_i$

$$\overline{a} < 0$$

Negative acceleration is also called deceleration or retardation.

Average Acceleration:

Total change in velocity divided by total time is called average acceleration.

$$\langle \overline{a} \rangle = \frac{\text{Total change in velocity}}{\text{Total time}}$$

$$\langle \overline{a} \rangle = \frac{\Delta \overline{v}}{\Delta t}$$

Uniform Acceleration:

Equal changes in velocity in equal intervals of time is called uniform acceleration.

Instantaneous Acceleration:

The acceleration of a body at a particular instant of time is called instantaneous acceleration.

Mathematically:

$$\overline{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \overline{v}}{\Delta t}$$

When a body moves with uniform acceleration the average and instantaneous acceleration will be equal.

Q3. Prove equations of motion for uniform acceleration. Also define gravitational acceleration g.

Ans. First Equation of Motion:

$$v_f = v_i + at \quad \text{is called first equation of motion.}$$

Proof:

Consider a body is moving with initial velocity $\overline{v}_i$ in time "t" its velocity changes into final velocity v_f having uniform acceleration "a".

As

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$\overline{a} = \frac{\Delta \overline{v}}{\Delta t}$$

$$\overline{a} = \frac{\overline{v}_f - \overline{v}_i}{t_f - t_i}$$

$$\text{If initial time } t_i = 0$$

$$\Rightarrow \bar{a} = \frac{v_f - v_i}{t}$$

$$\Rightarrow \bar{a}t = \bar{v}_f - \bar{v}_i \quad \text{OR}$$

$$v_f = v_i + \bar{a}t$$

Required proof

Second Equation of Motion:

Consider a uniformly accelerated body covers distance "S".

AS $s = \langle v \rangle t$ ----- (i)

Also $\langle v \rangle = \frac{v_f + v_i}{2}$ ----- (ii)

Put (ii) in (i) $s = \left(\frac{v_f + v_i}{2} \right) t$ ----- (iii)

Put $v_f = v_i + at$

$$s = \left(\frac{v_i + at + v_i}{2} \right) t$$

$$s = \left(\frac{2v_i + at}{2} \right) t$$

$$s = \left(\frac{2v_i}{2} + at \right) t$$

$$s = v_i t + \frac{1}{2} at^2 \quad \text{..... Proof}$$

Third Equation of Motion:

Consider a body is uniformly accelerated and covers distance "S".

As $s = \langle \bar{v} \rangle t$ ----- (i)

Also $\langle \bar{v} \rangle = \frac{v_f + v_i}{2}$ ----- (ii)

Put (ii) in (i) $s = \left(\frac{v_f + v_i}{2} \right) t$ ----- (iii)

As $v_f = v_i + at$

OR $t = \frac{v_f - v_i}{a}$

Put $t = \frac{v_f - v_i}{a}$ in (iii)

$$s = \left(\frac{v_f + v_i}{2} \right) \cdot \left(\frac{v_f - v_i}{a} \right)$$

$$S = \frac{v_f^2 + v_i^2}{2a}$$

Use $a^2 - b^2 = (a + b)(a - b)$

OR

$$2as = v_f^2 - v_i^2$$

Required proof

Gravitational Acceleration "g":

The acceleration produced in freely falling bodies or due to gravitational field of earth is called gravitational acceleration.

Symbol:

Its symbol is g .

Value:

In absence of air near earth surface its value is 9.8 m/s^2 or 980 cm/s^2 or 32 ft/s^2 . If a body is moving upward its gravitational acceleration is taken negative while for a body falling (moving) downward is taken positive.

Equations of Motion for Freely Falling Bodies:

$$v_f = v_i + at \Rightarrow v_f = v_i + gt$$

$$s = v_i t + \frac{1}{2} at^2 \Rightarrow h = v_i t + \frac{1}{2} gt^2$$

$$2as = v_f^2 - v_i^2 \Rightarrow 2gh = v_f^2 - v_i^2$$

Q4. Define and explain impulse.

Ans. Impulse:

The phenomenon (عمل) in which very large force acts on a body for a very short interval of time.

OR

The product of force $\overline{F}$ and time interval is called impulse.

Mathematically:

$$J = \overline{F} \times \Delta t$$

Nature:

Impulse is a vector quantity.

Unit:

The unit of impulse is N.S.

Dimensions:

The dimensions of impulse and momentum are same i.e. $[MLT^{-1}]$.

Example:

- i) Hitting a cricket ball
- ii) Hammering
- iii) Blowing or striking etc.

To Show Impulse is Change in Momentum:

As $J = \overline{F_{av}} \times \Delta t$

But $\overline{F}_{ave} = \frac{\Delta \overline{P}}{\Delta t}$ (Second law)

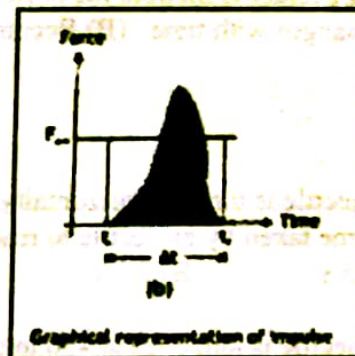
Put in above equation.

$$\overline{J} = \frac{\Delta \overline{P}}{\Delta t} \times \Delta t$$

$$\overline{J} = \Delta \overline{P} \quad \text{OR}$$

$$\overline{J} = \overline{P}_f - \overline{P}_i$$

This shows impulse is change in momentum.



MCO's

1. A ball is thrown vertically upward at 19.6 m/s. For its complete trip (up and back down) to the starting position, its average speed is.....
(A).19.6 m/s (B).9.8 m/s (C).6.5 m/s (D).4.9 m/s
2. If you throw a ball downward, then its acceleration immediately after leaving hand, assuming no air resistance, is.....
(A).9.8 m/s² (B).more than 9.8 m/s² (C).less than 9.8 m/s²
(D).speed of throw is required for answer
3. The time rate of change of momentum gives.....
(A).Force (B). impulse (C).Acceleration (D).Time
4. The area between velocity-time graph is numerically equal to.....
(A).Velocity (B).Displacement (C).Acceleration (D).Time
5. If the slope of velocity –time graph gradually decreases, then the body is said to be moving with.....
(A).Positive acceleration (B).negative acceleration
(C).uniform velocity (D).zero acceleration
6. A 7.0 kg bowling ball experiences a net force of 5.0 N. What will be its acceleration?
(A).35 m/s² (B).7 m/s² (C).5 m/s² (D).0.71 m/s²
7. S.I unit of impulse is.....
(A).kg m/s² (B).N s (C).N/s (D).N m
8. A ball with original momentum +4.0 kg m/s hits a wall and bounces straight back without losing any kinetic energy. The change in momentum of the ball is.....
(A).+4 N s (B).-4 N s (C).+8 N s (D).-8 N s
9. A body is traveling with a constant acceleration of 10 m/s². If S₁ is the distance traveled in 1st second and S₂ is the distance traveled in 2nd second, which of the following shows a correct relation between S₁ and S₂?
(A).s₁ = s₂ (B).s₁ = 3s₂ (C).s₂ = 3s₁ (D).2s₂ = 3s₁

horizontal component of velocity.....
 changes with time (B).Becomes Zero (C).Remains constant (D).Increases with time

11. A projectile is thrown horizontally from a 490 m high cliff with a velocity of 100 m/s. The time taken by projectile to reach the ground is.....
 (A).2.5 s (B).5.0 s (C).7.5 s (D).10 s
12. A projectile is launched at 45° to the horizontal with an initial kinetic energy E. Assuming air resistance to be negligible, what will be the kinetic energy of the projectile when it reaches its highest point?
 (A).0.50 E (B).0.71 E (C).0.70 E (D).E
13. To improve the jumping record, the long jumper should jump at an angle of.....
 (A).30° (B).45° (C).60° (D).90°
14. Range of a projectile on a horizontal plane is same for the following pair of angles.
 (A).15° and 18° (B).43° and 47° (C).20° and 80°
 (D).52° and 62°

Answer Keys:

1	B	5	B	9	C	13	B
2	A	6	D	10	C	14	B
3	A	7	B	11	D		
4	B	8	D	12	A		

Solution of MCQs

1. $v_i = 19.6 \text{ m/s}$ $v_{ave} = \frac{v_i + v_f}{2}$
 $v_f = 0 \text{ m/s}$ $v_{ave} = \frac{0 + 19.6}{2}$
 $v_{ave} = ?$
 Ans. **B. 9.8 m/s^2**

2. When the ball leaves your hand, the only force acting on the ball is gravitational force (neglecting air resistance) and the acceleration due to gravity is "g" which is equal to 9.8 m/s^2 .
 So. Ans. **A. 9.8 m/s^2**

3. Ans. **A. Force** $\bar{F} = \frac{\Delta P}{\Delta t}$

4. Ans. **B. Displacement**

5. Ans. **B. -ive acceleration**

6. $m = 7\text{kg}$ $\bar{F} = ma$ $a = \frac{5}{7}$

$\bar{F} = 5\text{N}$ $\bar{a} = \frac{\bar{F}}{m}$ $\bar{a} = 0.71\text{m/s}^2$

$\bar{a} = ?$ Ans. **D. 0.71m/s^2**

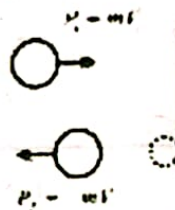
7. $J = F_{\text{ave}} \Delta t$ Impulse unit = N.S

Ans. **B. NS**

8. $\Delta P = mv_f - mv_i = P_f - P_i = -mv - mv = -2mv = -2P$

$\Delta P = -2(4) = -8\text{NS}$

Ans. **D. -8NS**



9. Distance covered in nth second is:

$$s_n = v_i + \frac{a}{2}(2n-1)$$

For $n = 1$ $s_1 = 0 + \frac{10}{2}(2(1)-1)$

$$s_1 = 5(2-1) = 5\text{m}$$

For $n = 2$ $s_2 = 0 + \frac{10}{2}(2(2)-1)$

$$s_2 = 5(4-1) \quad s_2 = 5(3)$$

$$s_2 = 15$$

$$s_2 = 3 \times 5$$

$$s_2 = 3s_1$$

Ans. **C. $s_2 = 3s_1$**

10. Ans. **C. Remains constant**

11. $v_i = 100\text{m/s}$, $v_f = 0\text{m/s}$ $g = 9.8\text{m/s}^2$

$$v_f = v_i + gt$$

$$0 = 100 - 9.8t$$

$$t = \frac{100}{9.8} = 10.2 \text{ s}$$

 Ans. **D. 10S**

12.

Velocity at initial point

$$v^2 = v_{ix}^2 + v_{iy}^2$$

$$v^2 = (v_i \cos \theta)^2 + (v_i \sin \theta)^2$$

$$v^2 = v_i^2 \cos^2 \theta + v_i^2 \sin^2 \theta$$

$$v^2 = v_i^2 (\cos^2 \theta + \sin^2 \theta)$$

$$v^2 = v_i^2 \dots \dots \dots (1)$$

$$v^2 = v_i^2$$

So at initial point is

$$KE = \frac{1}{2} m v^2$$

$$E = \frac{1}{2} m v_i^2$$

Velocity at highest point

$$v^2 = v_{fx}^2 + v_{fy}^2$$

$$\text{As } v_{fx} = v_{ix} = v_i \cos \theta$$

$$\text{And } F_{fy} = 0$$

$$\text{So } v^2 = v_{fx}^2 + 0$$

$$v^2 = (v_i \cos \theta)^2 = v_i^2 \cos^2 \theta$$

So KE at highest point is:

$$KE = \frac{1}{2} m v_i^2 \cos^2 \theta$$

$$KE = \left(\frac{1}{2} m v_i^2 \right) \cos^2 \theta$$

$$KE = E \cos^2 \theta$$

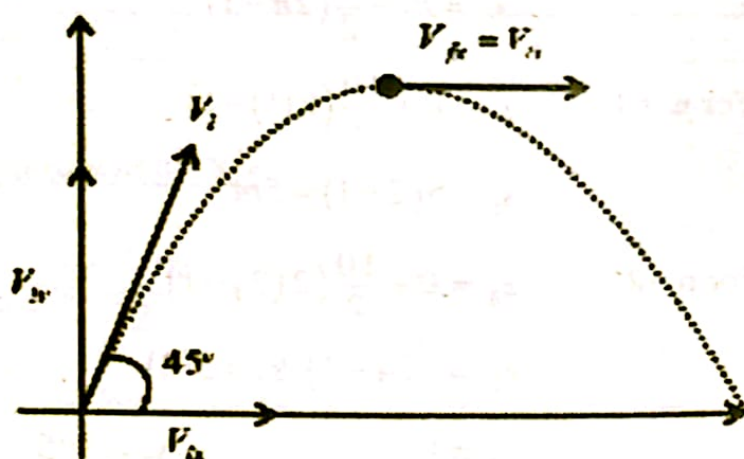
$$KE = E (\cos 45^\circ)^2$$

$$(KE)_H = E \left(\frac{1}{\sqrt{2}} \right)^2$$

$$(KE)_H = E \left(\frac{1}{2} \right)$$

$$(KE)_H = 0.5 E$$

Therefore

 Ans. **A. 0.5 E**


13.

Ans.

B. 45°

14.

 As $43 + 47 = 90^\circ$

So they are complementary angles

we know that for the same range of

projectile the angles must be complementary angles.

Ans.

$$B. 43^\circ + 47^\circ$$

Short Questions & Answers

Q.1: If you are riding on a train, that train speed past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward why?

Ans. Thesis Statement:

The other train on an adjacent track appears to be moving backward because of,

- i) Your frame of reference and
- ii) Relativity of motion

Relativity of Motion:

The relativity of motion means that motion is a relative phenomenon and is not absolute.

- i) A relative phenomenon is one that depends on the frame of reference of the observer.
- ii) Thus a body may be at rest for one observer but in motion for another observer depending upon their frame of references.

In the Current Case:

- i) Your frame of reference is the train that you are riding on from where you are observing the other train.
- ii) If the train you are riding on is traveling with a uniform velocity, you will consider it at rest.
- iii) Since your train (which is your frame of reference) is moving faster than the other train on the adjacent track, it appears to you that the other train is moving backward.

Conclusion:

Thus it is concluded that the other train appears to be moving backward due to your frame of reference and relativity of motion.

Q.2: Can the velocity of a body reverse direction, when acceleration is constant? If you think so give an example.

Ans. Thesis Statement:

Yes, the velocity of a body can reverse its direction when acceleration is constant.

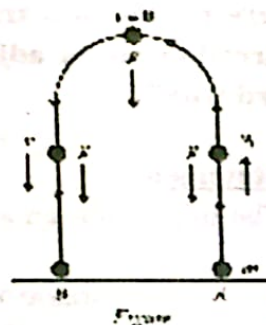
Example:

Consider a body thrown upward with an initial velocity " v_i ".

- i) During its upward motion its velocity is directed upward while the gravitational acceleration $g = 9.8 \text{ m/s}^2$ acting downwards, is constant.
- ii) The velocity of the body decreases with the passage of time and at its height point it becomes zero for a moment and then reverses its direction and starts moving downward.
- iii) The gravitational acceleration "g" is still acting downward. Which means ($g = 9.8 \text{ m/s}^2$) remains constant although the velocity has reversed its direction.

Figurative Interpretation:

From the figure it is clear that ($g = 9.8 \text{ m/s}^2$) is directed downward through the path from "A" to "B" and the velocity has reversed its direction from upward to downward at the peak point.



Conclusion:

Thus it is concluded that the velocity of a body can reverse its direction when the acceleration is constant. Projectile motion is example.

Q.3: When you stand still on the ground, how large a force does the ground exerts on you? Why doesn't the force make you rise into the air?

Ans. Thesis Statement:

When you stand still on the ground, the ground exerts a force on you, equal to the weight of your body.

Action and Reaction:

According to Newton's 3rd law of motion, when you are standing on the ground, you are exerting a downward force on the ground which is equal to your weight.

$$F_{\text{action}} = w = mg$$

This is the action force.

- i) As a result the ground exerts an equal amount of force on your body in the opposite direction this is the reaction force.

$$F_{\text{reaction}} = -F_{\text{action}}$$

OR

$$F_{\text{reaction}} = -(mg)$$

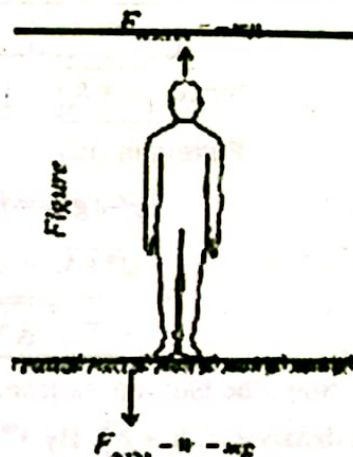
$$\therefore F_{\text{action}} = w = mg$$

- ii) The negative sign shows that action and reaction are always opposite in direction.

The Force Doesn't Make You Rise in Air:

As can be seen from the figure:

- i) The reaction force is only equal to the action (weight= mg) and can never exceed the action force.
- ii) Since both are acting in opposite direction, and thus balances each other. That is why this doesn't make you rise into the air.

**Conclusion:**

Thus it is concluded that the ground exerts a force on you, equal to your weight (mg). This upward force is neutralized by the weight of your body and does not make you rise into the air.

Q4. A man standing on the top of a tower throws a ball vertically up with certain velocity. He also throws another ball vertically down with the same speed. Neglecting air resistance which ball will hit the ground with higher speed?

Ans. Thesis Statement:

Both the balls will hit the ground exactly with the same speed but at different times.

Explanation:

Consider a man standing on the top of a tower with two balls B_1 and B_2 in his hand.

Ball B_1 Thrown Downward:

- The man throws B_1 vertically downward with velocity " V_i ".
- The final velocity of the ball B_1 is given by:

$$2as = v_f^2 - v_i^2 \text{ ----- (i)}$$

$$\Rightarrow 2gh = v_f^2 - v_i^2 \quad \therefore \boxed{s = h} \quad \text{And} \quad \boxed{a = g}$$

$$2gh + v_i^2 = v_f^2$$

OR $v_f^2 = 2gh + v_i^2$

$$\sqrt{v_f^2} = \sqrt{2gh + v_i^2}$$

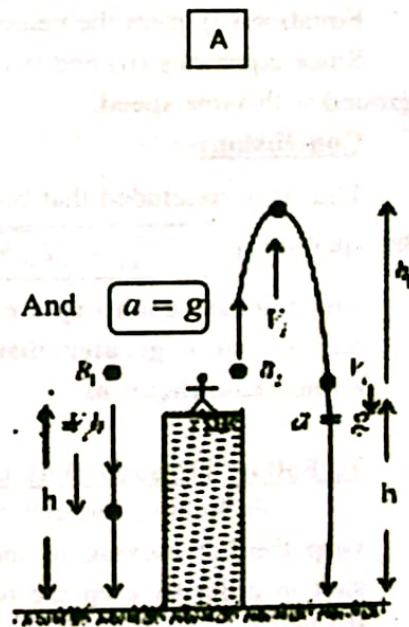
$$\boxed{v_f = \sqrt{2gh + v_i^2}} \text{ ----- (ii)}$$

The ball B_1 will hit the ground with this velocity.

Ball B_2 Thrown Upward:

The man throws ball B_2 with the same initial velocity " V_i ".

- It covers distance h_1 , at the peak its velocity is zero.
By 3rd equation of motion



$$2as = v_f^2 - v_i^2 \text{----- (iii)}$$

Since $s = h_1$, $a = -g$ and $v_f = 0$

Putting in (iii)

$$\cancel{2}gh_1 = (0)^2 \cancel{-} v_i^2$$

$$\sqrt{2gh_1} = \sqrt{v_i^2}$$

$$v_i = \sqrt{2gh_1} \text{----- (iv)}$$

Now, the ball moves from point "A" (peak) to the ground its initial velocity is zero & distance = $(h_1 + h)$. By 3rd equation of motion:

$$2as = v_f^2 - v_i^2$$

$$\therefore v_i = 0 \text{ , } a = g \text{ and } s = (h_1 + h)$$

Putting the values,

$$2g(h_1 + h) = v_f^2 - (0)^2$$

$$2gh_1 + 2gh = v_f^2$$

$$\text{OR } \sqrt{v_f^2} = \sqrt{2gh_1 + 2gh} \quad \therefore 2gh_1 = v_i^2 \text{ from eqn (iv)}$$

$$\text{So } v_f = \sqrt{v_i^2 + 2gh}$$

$$\text{OR } v_f = \sqrt{2gh + v_i^2} \text{----- (v)}$$

Equation (v) gives the velocity with which ball "B₂" will hit the ground.

Since equations (ii) and (v) are exactly the same, which means both the balls will hit the ground with same speed.

Conclusion:

Thus it is concluded that both the balls will hit the ground with the same speed given by the equation; $v_{f(\text{hitting})} = \sqrt{2gh + v_i^2}$.

Q5. The cricket coach explains that the follow-through with the shot will make the ball travel a greater distance. Explain the reason in terms of the impulse-momentum theorem.

Ans. To Follow Through With the Shot:

Following through the shot means to keep the bat moving in the direction of the shot in order to keep the bat in contact with the ball for longer time.

Bat & ball remains in touch for long time.

Impulse-Momentum Theorem:

The impulse-momentum theorem is given by:

$$J = \bar{F}_{ave} \Delta t = \Delta \bar{P} \text{----- (i)}$$



$\bar{F}_{ave}$ is the applied force

Δt is the time for which force $\bar{F}_{ave}$ is applied

ΔP is change in momentum of the body (ball)

Explanation:

By following through the shot will increase the time of contact (Δt) between bat and the ball. Since Δt is directly proportional to ΔP (change in momentum) of the ball. Thus increasing Δt will increase the distance travelled by the ball.

As $F \Delta t = \Delta P$
 $\therefore \Delta P = m \Delta v$

Keep F and m constant

Therefore $F \Delta t = m \Delta v$
 $\Delta t \propto \Delta v$

Greater velocity means longer distance will be travelled.

Conclusion:

Thus increasing the time of contact between bat and the ball (by following through with shot) will increase the distance travelled by the ball.

Q6. When you release an inflated but untied balloon, why does it fly across the room?

Ans. Inflated Balloon Flies Across the Room:

When you release an inflated but untied balloon it flies across the room because of

- Newton's third law and
- Conservation of momentum

Explanation:

The gas inside a balloon rushes out of the opening with high speed due to high pressure inside.

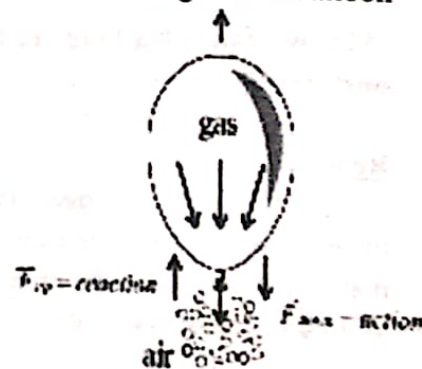
- This gas exerts a force on the surrounding air and pushes it backwards which is the action force.
- In response to this force the surrounding air exerts an equal amount of force on the balloon in the forward-direction.
- Due to the reaction force on balloon by the surrounding air the balloon flies across the room.

Conservation of Momentum:

- When the gas rushes out of the balloon the escaping gas has mass " m_g " and velocity " v_g " and thus it has backward momentum

$$\bar{P} = m_g v_g$$

- To conserve the momentum the balloon along with the remaining gas



will start moving in opposite direction i.e.

$$\Delta P_{\text{forward}} = m_B v_B$$

v_B = Velocity of Balloon
 m_B = Mass of Balloon

Thus the balloon will start flying across the room.

Conclusion:

Due to the conservation of momentum the balloon flies across the room when released.

Q7. Modern cars are not rigid but are designed to have “crumple zones” (irregular fold) that collapse upon impact. What is the advantage of this new designed?

Ans. Modern cars are not rigid but are designed to have “crumple zones” that collapse upon impact.

Crumple Zones in Modern Cars:

- Crumple zones or crush zones are areas of a car that are designed to deform and crumple in a collision. This absorbs some of the energy of the impact, preventing it from being transmitted to the occupants.
- These structural safety features are used in modern cars to increase the time during which change in velocity (and as a result in momentum) occurs from the impact during collision by controlled deformation.

Advantages of Crumple-Zones:

Some of the advantages of the “crumple-zones” are given below:

- They allow the car to crush in a controlled way in a collision.
- They increase the time to change the momentum of the driver and passengers in a crash, which reduces the force involved.
- The average force applied to the occupants is inversely related to the time over which it is applied, as:

$$F = \frac{\Delta P}{\Delta t}$$

$$\Rightarrow F = \frac{m \Delta v}{\Delta t}$$



- Crumple zones are located in the front part of the cars, in order to absorb the impact of a head-on collision.



Q8. Why we can hit a long six in a cricket match rather than if we toss a ball for ourselves?

Ans. Reason:

We can hit a longer six in a cricket match than if we toss (ہوا میں اچھالنا) a ball to ourselves because, the momentum of the ball thrown by the bowler in a cricket match is greater than the momentum of the ball we toss to ourselves (خود اپنے لئے گیند) (کو ہوا میں اچھال کر چھکا لگنا).

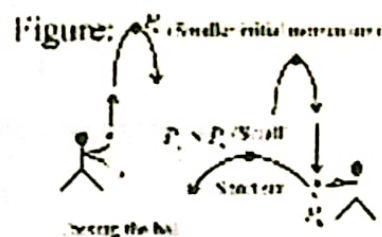
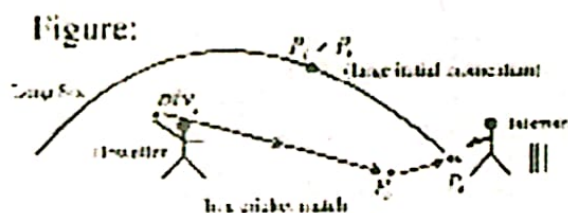
Explanation:

The final momentum " P_f " of the ball, after hit by the bat is equal to the sum of the initial momentum of the ball " P_b " and the bat " P_B " i.e.

$$P_f = P_b + P_B \text{ ----- (i)}$$

If we keep the momentum of the bat constant, the final momentum " P_f " of the ball is directly proportional to the initial momentum " P_b " of the ball $P_f \propto P_b$.

- Since the initial momentum " P_b " of the ball in a cricket match is greater (velocity of ball is high) as compared to the momentum of ball " P_b " if we toss the ball to ourselves.
- Therefore a longer six can be hit in a match than if we toss a ball to ourselves.

**Conclusion:**

We can hit a long six in a cricket match rather than if we toss a ball for ourselves due to the greater momentum of the ball bowled by a bowler in a match compared to the momentum of the ball that we toss to ourselves.

Q9. An aeroplane, while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.

Ans. Reason:

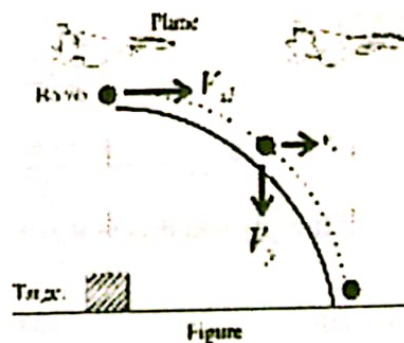
An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target due to the horizontal components of its velocity (inertia).

Explanation:

Initially the bomb in the plane has exactly the same horizontal velocity as that of the plane " V_x ".

Inertia of the Bomb: As soon the bomb is dropped, it started moving downwards due to gravity with velocity " V_y ".

- But due to inertia the bomb will continue its horizontal motion with velocity " V_x ". Therefore bomb will move along a parabola and will hit the ground away from the target.



Calculating the Angle of Projection:

The K.E of a projectile at the point of projection "0" is given by:

$$K.E_0 = \frac{1}{2} m v_i^2 \text{ ----- (i)}$$

- At the summit (point A) the vertical component is zero i.e. $v_{iy} = 0$ and the horizontal component is $v_i \cos \theta$

Thus the K.E at the summit is given by:

$$K.E_A = \frac{1}{2} m (v_i^2 \cos^2 \theta)$$

$$K.E_A = \frac{1}{2} m v_i^2 \cos^2 \theta \text{ ----- (ii)}$$

According to the condition of the question

$$K.E_A = \frac{1}{4} (K.E_0) \text{ ----- (iii)}$$

Putting values from (i) and (ii) in equation (iii)

$$(iii) \Rightarrow \frac{1}{2} m v_i^2 \cos^2 \theta = \frac{1}{4} \left(\frac{1}{2} m v_i^2 \right)$$

$$\Rightarrow \sqrt{\cos^2 \theta} = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1} \left(\frac{1}{2} \right) \Rightarrow$$

$$\theta = 60^\circ$$

Conclusion:

Thus the angle of projection is 60° for a projectile, for which it's K.E at the summit is equal to one-fourth of the K.E at the point of projection.

Q11. For any specific velocity of projection, the maximum range is equal to four times of the corresponding height. Discuss.

Ans. Thesis Statement:

For any specific velocity of projection the maximum range is equal to four times of the corresponding height, because the maximum horizontal range of a projectile is given by:

$$R_{\max} = \frac{v_i^2 \sin 2\theta}{g} \text{ ----- (i)}$$

$$R_{\max} = \frac{v_i^2 2 \sin \theta \cdot \cos \theta}{g} \text{ ----- (ii)}$$

$$\therefore \sin 2\theta = 2 \sin \theta \cdot \cos \theta$$

Multiply and divide R.H.S of equation (ii) by $\left(\frac{\sin \theta}{2} \right)$

$$(ii) \Rightarrow R_{\max} = \frac{v_i^2 2 \sin \theta \cdot \cos \theta}{g} \times \frac{\sin \theta / 2}{\sin \theta / 2}$$

$$\Rightarrow R_{\max} = \frac{4v_i^2 \sin^2 \theta \cos \theta}{2g \sin \theta}$$

$$\Rightarrow R_{\max} = 4 \frac{\cos \theta}{\sin \theta} \left(\frac{v_i^2 \sin^2 \theta}{2g} \right) \text{----- (iii)}$$

Since $\left(\frac{v_i^2 \sin^2 \theta}{2g} \right) = (H)$ Put in equation (iii)

$$(iii) \Rightarrow R_{\max} = 4 \frac{\cos \theta}{\sin \theta} (H)$$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$\Rightarrow R_{\max} = 4 \cot \theta (H)$$

$$\Rightarrow R_{\max} = 4 \frac{1}{\tan \theta} (H)$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\Rightarrow R_{\max} \times \tan \theta = 4 (H) \text{----- (iv)}$$

We know that range of projectile is maximum at 45° i.e. $\theta = 45^\circ$

Put in equation (iv)

$$\Rightarrow R_{\max} \times \tan (45^\circ) = 4 (H) \text{----- (iv)}$$

$$\therefore \tan 45^\circ = 1$$

$$\Rightarrow R_{\max} \times 1 = 4H \text{----- (v)}$$

Conclusion: Thus equation (v) implies that the maximum range of projectile ($R_{\max}$) is equal to four times the corresponding height. ($4H$).

Q12. What is the angle for which the maximum height reached and corresponding range are equal?

Ans. Thesis Statement:

The angle for which the maximum height reached and corresponding range are equal is 76° .

Calculating the Angle:

According to the condition of the question maximum height is equal to the horizontal range i.e.

$$H_{\max} = R \text{----- (i)}$$

Putting the values of " $H_{\max}$ " and " R " in equation (i)

$$\Rightarrow \frac{v_i^2 \sin^2 \theta}{2g} = \frac{v_i^2 \sin 2\theta}{g}$$

$$\Rightarrow \frac{\sin^2 \theta}{2} = 2 \sin \theta \cos \theta$$

$$\therefore \sin^2 \theta = 2 \sin \theta \cos \theta$$

$$\Rightarrow \sin \theta = 4 \cos \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = 4$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\Rightarrow \tan \theta = 4$$

$$\Rightarrow \theta = \tan^{-1}(4) = 76^\circ$$

$$\Rightarrow \theta = 76^\circ$$

Required Angle

Conclusion: Thus the angle for which the maximum height reached and corresponding range are equal is $\theta = 76^\circ$

Since

$$H_{\max} = \frac{v_i^2 \sin^2 \theta}{2g}$$

&

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

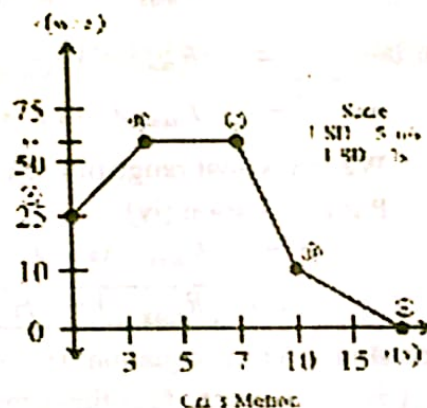
Assignment Solution**Assignment 3.1:**

The velocity time graph shows the motion of car in a straight line. By reading the scale carefully, calculate (a) the acceleration of the car between segment A and B, B and C, C, D and D and E, from the slope of the graph. Also (b) Calculate the car's average acceleration for the complete journey.

(8.33 m/s², 0 m/s², -13.33 m/s²,
-1.67 m/s² and -1.56 m/s²)

Given Figure As:Required:

- Calculate acceleration
From A to B
From B to C
From C to D
From D to E
- Calculate average acceleration of car for complete journey.



Solution (a): The acceleration from point A to B can be calculated by measuring the slope as:

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{55 - 25}{3 - 0} = \frac{30}{3} = 10 \text{ m/sec}^2$$

Therefore

$$a = 10 \text{ m/sec}^2 \text{Answer}$$

The acceleration from point B to C by measuring the slope is:

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{55 - 55}{7 - 3} = \frac{0}{4} = 0 \text{ m/sec}^2$$

Therefore

$$a = 0 \text{ m/sec}^2 \text{Answer}$$

The acceleration from point C to D by measuring the slope is:

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{10 - 55}{10 - 7} = \frac{-45}{3} = -15 \text{ m/sec}^2$$

Therefore

$$a = -15 \text{ m/sec}^2 \text{Answer}$$

The acceleration from point D to E by measuring the slope is:

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 - 10}{16 - 10} = \frac{-10}{6} = -1.67 \text{ m/sec}^2$$

Therefore

$$\bar{a} = -1.67 \text{ m/sec}^2$$

Solution (b): The average acceleration can be calculated by measuring the slope from point A to E as:

$$\bar{a}_{ave} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 - 25}{16 - 0} = \frac{-25}{16} = -1.56 \text{ m/sec}^2$$

$$\bar{a}_{ave} = -1.56 \text{ m/sec}^2$$

Assignment.3.2:

A proton moving with a speed of $1.0 \times 10^7 \text{ ms}^{-1}$ passes through a 0.020 cm thick Sheet of paper and emerges with a speed of $2.0 \times 10^6 \text{ ms}^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Given Data:

Initial speed of proton $= v_i = 1.0 \times 10^7 \text{ m/s}$

Final speed of proton $= v_f = 2.0 \times 10^6 \text{ m/s}$

Distance covered $= s = 0.02 \text{ cm} = 0.02 \times 10^{-2} \text{ m}$

Required Data:

a) Retardation $= a = ?$

b) Time taken $= t = ?$

Solution (a): We know that

$$2as = v_f^2 - v_i^2$$

$$2a(0.02 \times 10^{-2}) = (2 \times 10^6)^2 - (1.0 \times 10^7)^2$$

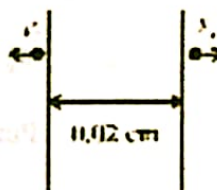
$$0.0004 \times a = 4 \times 10^{12} - 1 \times 10^{14} = 4 \times 10^{12} - 100 \times 10^{12}$$

$$0.0004a = (4 - 100) \times 10^{12}$$

$$0.0004a = -96 \times 10^{12}$$

$$a = \frac{-96}{0.0004} \times 10^{12} \text{ m/s}^2$$

$$a = -2.4 \times 10^{17} \text{ m/sec}^2$$



Solution (b): We know that

$$V_f = V_i + at$$

$$\Rightarrow V_f - V_i = at$$

$$\Rightarrow t = \frac{V_f - V_i}{a}$$

$$\Rightarrow t = \frac{2 \times 10^6 - 1 \times 10^7}{-2.4 \times 10^{17}} = \frac{-8 \times 10^6}{-2.4 \times 10^{17}} = 3.3 \times 10^{-11} \text{ sec}$$

$$\Rightarrow t = 3.3 \times 10^{-11} \text{ sec}$$

Assignment 3.3:

Suppose that the mass of the spacecraft 'm,' is 11000 kg and that the mass of the astronaut 'm,' is 92 kg. In addition, assume that the astronaut pushes with a force of $F = +36 \text{ N}$ (along x-axis) on the spacecraft. Find the accelerations of the spacecraft and the astronaut.

Given Data:

$$\text{Mass of spacecraft} = m_s = 11000 \text{ kg}$$

$$\text{Mass of Astronaut} = m_A = 92 \text{ kg}$$

$$\text{Force} = F = 36 \text{ N} \quad \text{along x-axis}$$

Required Data:

$$\text{Acceleration of the spacecraft} = a_s = ?$$

$$\text{Acceleration of the Astronaut} = a_A = ?$$

Solution:

We know that

$$F = ma$$

$$a = \frac{F}{m}$$

$$a_s = \frac{F}{m_s}$$

Putting values

$$a_s = \frac{36}{11000} = 0.0033$$

$$a_s = 0.0033 \text{ m / sec}^2$$

Also

$$a_A = \frac{F}{m_A}$$

Putting values

$$a_A = \frac{36}{92} = 0.39 \text{ m / sec}^2$$

$$a_A = -0.39 \text{ m / sec}^2$$

Assignment 3.4:

A girl of mass 48.0 kg is rescued from a building fire by leaping into a firefighters' net. The window from which she leapt was 12.0 m above the net. She lands in the net so that she is brought to a complete stop in 0.45 s. During this interval (a) What is his change in momentum? (b) What is the impulse on the net due to the girl? (c) What is the average force on the net due to the girl?

(a) $11.2 \times 10^3 \text{ kg m s}^{-1}$ (UP) (b) $11.5 \times 10^3 \text{ kg m s}^{-1}$ (DOWN)

(c) 25.6 A0' kg m s' (DOWN)

Given Data:

$$\text{Mass of girl} = m = 48.0 \text{ kg}$$

$$\text{Height of window from net} = h = 12.0 \text{ m}$$

$$\text{Initial velocity of girl} = v_i = 0 \text{ m/sec}$$

$$\text{Time of impact} = \Delta t = 0.45 \text{ sec}$$

Required Data:

- Change in momentum = $\Delta P = ?$
- Impulse on the net due to girl = $J = ?$
- Average force on net due to girl = $F_{\text{ave}} = ?$

Solution (a):

To calculate the final velocity of the girl just before hitting the net we have:

$$2as = v_f^2 - v_i^2 \quad (s = h, a = g)$$

Putting values

$$2(9.8)(12) = v_f^2 - 0^2$$

$$v_f^2 = 235.2 \Rightarrow \sqrt{v_f^2} = \sqrt{235.2}$$

$$\Rightarrow v_f = 15.3 \text{ m/sec}$$

Now change in momentum is given by:

$$\Delta P = P_f - P_i = mv_f - mv_i$$

Putting values

$$\Delta P_i = (48)(15.34) - (48)(0)$$

$$\Delta P_i = 736.14 - 0 = 736.14 \text{ kg m/sec}$$

This momentum will act as an initial momentum of the girl just before hitting the net i.e. $P_i = \Delta P_i = 736.14 \text{ kg m/sec}$

When the girl hit the net and come to rest her final momentum will become zero i.e. $P_f = 0 \text{ kg m/s}$

$$\text{Therefore } \Delta P = P_f - P_i = 0 - 736.14 = -736.14 \text{ kg m/sec}$$

The negative sign shows that the change in momentum is opposite in direction to the motion i.e. change in momentum is upward.

So we can write

$$\Delta P = 736.14 \text{ N sec (up)} \dots \dots \dots \text{Required Answer}$$

Solution (b):

Now to calculate the impulse on the net by the girl:

We know that

$$J = mg\Delta t + \Delta P \quad (\text{Gravity cannot be ignored in impulse})$$

Putting values

$$J = (48)(9.8)(0.45) + 736.14$$

$$J = 211.68 + 736.14 = 947.82 \text{ kg m/sec}$$

$$\bar{J} = 947.82 \text{ NS (down)} \dots\dots\dots \text{Answer}$$

Solution (c):

To calculate average force we are using the relation

$$J = F_{ave} \times \Delta t \Rightarrow F_{ave} = \frac{J}{\Delta t}$$

Putting values

$$F_{ave} = \frac{947.82}{0.45}$$

$$F_{ave} = 2106.27 \text{ N (down)} \dots\dots\dots \text{Answer}$$

Assignment 3.5:

On a highway a car of mass 1500 kg is stopped at traffic signal. A pickup of mass 2000 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0 m/s. How fast was the pickup going just before the collision?

(8.75 m/s (31.5 km/hr))

Given Data:

Mass of car = $m_1 = 1500 \text{ kg}$

Velocity of car before collision $u_1 = 0 \text{ m/s}$

Mass of pickup = $m_2 = 2000 \text{ kg}$

Velocity of car after collision $= v_1 = 10 \text{ m/s}$

**Required Data:**

Velocity of pickup before collision $= u_2 = ?$

Solution:

We know that

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2$$

$$u_2 \left(\frac{2m_2}{m_1 + m_2} \right) = v_1 - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1$$

$$u_2 = \left[v_1 - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 \right] \left(\frac{m_1 + m_2}{2m_2} \right)$$

Putting values we get

$$u_2 = \left[10 - \left(\frac{1500 - 2000}{1500 + 2000} \right) \times 0 \right] \left(\frac{1500 + 2000}{2(2000)} \right)$$

$$u_2 = (10) \left(\frac{3500}{4000} \right) = (10) \frac{35}{40} = \frac{35}{4} = 8.75 \text{ m/s}$$

$$u_2 = 8.75 \text{ m/s} \dots\dots\dots \text{Required Answer}$$

Now convert it into km / hr

$$u_2 = 8.75 \times \frac{1}{1000} \text{ km} \times \frac{3600}{\text{hr}}$$

$$u_2 = 31.5 \text{ km/hr} \dots\dots\dots \text{Required Answer}$$

NOTE

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m} = \frac{1}{1000} \text{ km}$$

$$1 \text{ hr} = 3600 \text{ sec}$$

$$1 \text{ sec} = \frac{1}{3600} \text{ hr}$$

Assignment 3.6: At Arbab Niaz Cricket Stadium Peshawar a batsman hits the shot at initial velocity of 28 m/s. If the boundary is 72 m from the batsman, will the ball cross the boundary for a six? If the angle with the horizontal is (a) 30° (b) 45° and (c) 70°. (Ignore air resistance)

(a) No, R = 69.2 m (b) Yes R 80 m (c) No, R = 51.4 m

Given Data:

Initial velocity of the ball = $v_i = 28 \text{ m/s}$

Boundary from the batsman = $S = 72 \text{ m}$

Projection angle = $\theta_1 = 30^\circ$

Projection angle = $\theta_2 = 45^\circ$

Projection angle = $\theta_3 = 70^\circ$

Required Data:

a) Range = $R_1 = ?$

b) Range = $R_2 = ?$

c) Range = $R_3 = ?$

Solution:

We know that $R = \frac{v_0^2 \sin 2\theta}{g}$

For $\theta_1 = 30^\circ$

$$R_1 = \frac{v_0^2 \sin 2\theta_1}{g}$$

$$R_1 = \frac{(28)^2 \sin 2 \times 30}{9.8}$$

$$R_1 = \frac{(784) \sin 60^\circ}{9.8}$$

$$R_1 = \frac{784 \times 0.866}{9.8}$$

$$R_1 = 69.28 \text{ m}$$

For $\theta_2 = 45^\circ$

$$R_2 = \frac{v_0^2 \sin 2\theta_2}{g}$$

$$R_2 = \frac{(28)^2 \sin 2 \times 45}{9.8}$$

$$R_2 = \frac{784 \sin 90^\circ}{9.8}$$

$$R_2 = \frac{784 \times 1}{9.8}$$

$$R_2 = 80 \text{ m}$$

For $\theta_3 = 70^\circ$

$$R_3 = \frac{v_0^2 \sin 2\theta_3}{g}$$

$$R_3 = \frac{(28)^2 \sin 2 \times 70}{9.8}$$

$$R_3 = \frac{(784) (\sin 140^\circ)}{9.8}$$

$$R_3 = \frac{784 \times 0.64}{9.8}$$

$$R_3 = 51.4 \text{ m}$$

No, the ball will not cross the boundary for a six.

Yes, the ball will cross the boundary for a six.

No, the ball will not cross the boundary for a six.

Numerical Problems

Problem-1:

An object is falling freely under gravity. How much distance will it travel in 2nd and 3rd second of its journey? (15m, 25 m)

Given Data:

$$g = 9.8 \text{ m/s}^2$$

$$v_i = 0 \text{ m/s}$$

Required Data:

Distance travel in 2nd second $s_2 = ?$

Distance travel in 3rd second $s_3 = ?$

Solution:

Formula for the distance traveled by the body in nth second is:

$$s_n = v_i + \frac{a}{2}(2n-1)$$

For 2nd Second:

i.e. $n = 2$

$$s_2 = v_i + \frac{9.8}{2}(2(2)-1)$$

$$s_2 = 0 + \frac{9.8}{2}(4-1)$$

$$s_2 = \frac{9.8}{2}(3) = 4.9(3) = 14.7 \quad 15\text{m}$$

$$s_2 = 15\text{m}$$

Now for 3rd Second: i.e. $n = 3$

$$s_3 = 0 + \frac{9.8}{2}(2(3)-1)$$

$$s_3 = 4.9(6-1)$$

$$s_3 = 4.9(5) = 24.5 \quad 25\text{m}$$

$$s_3 = 25\text{m} \dots\dots\dots \text{Required Answer}$$

A helicopter is ascending vertically at a speed of 19.6 ms^{-1} . When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

(8.0s)

Given Data:

Initial velocity of helicopter and stone $= v_i = 19.6 \text{ m/s}$

Acceleration due to gravity $= g = -9.8 \text{ m/s}^2$

Vertical distance covered by stone $= S = 156.8 \text{ m}$

Required Data:

Time taken by the stone to reach the ground $= t = ?$

Solution:

We know that

$$s = v_i t + \frac{1}{2} g t^2 \quad (a = g)$$

Putting values

$$156.8 = (19.6)t + \frac{1}{2}(-9.8)t^2$$

$$156.8 = 19.6t - 4.9t^2$$

$$156.8 = 4.9(4t - t^2)$$

$$32 = 4t - t^2$$

$$t^2 - 4t + 32 = 0$$

$$t^2 - 8t + 4t + 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t-8)(t+4) = 0$$

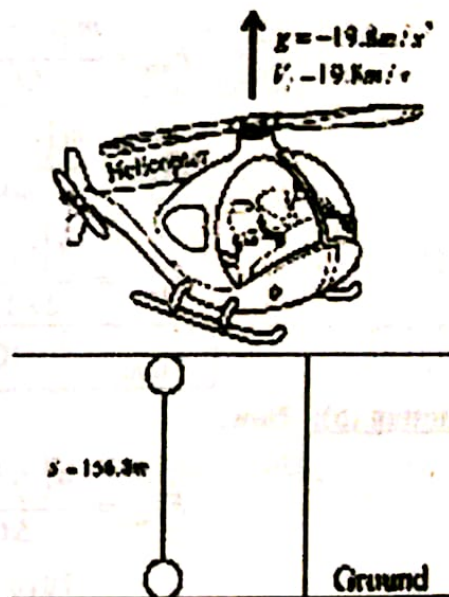
Either $t-8=0$ OR $t+4=0$

$$t = 8$$

$$t = -4$$

As time is taken as positive so we have

$$t = 8 \text{ sec}$$



Problem-3:

A car moving at 20.0 m/s (72.0 km/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 70 kg in each of the following cases. (a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 2.0 ms . (b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 45 ms , bringing the passenger to rest.

Given Data:

Initial velocity of the car $= V_i = 20 \text{ m/s}$

Final velocity of the car $= V_f = 0 \text{ m/s}$

Mass of the passenger $= m = 70 \text{ kg}$

Time interval $= \Delta t_1 = 2 \text{ ms} = 2 \times 10^{-3} \text{ sec}$

Time interval $= \Delta t_2 = 45 \text{ ms} = 45 \times 10^{-3} \text{ sec}$

Required Data:

a) Average force at $\Delta t_1 = F_{1 \text{ ave}} = ?$

b) Average force at $\Delta t_2 = F_{2 \text{ ave}} = ?$

Solution (a): As we know that

$$\Delta P = F_{\text{ave}} \Delta t$$

$$m \Delta v = F_{\text{ave}} \Delta t$$

$$m(v_f - v_i) = F_{\text{ave}} \Delta t$$

$$F_{\text{ave}} = \frac{m(v_f - v_i)}{\Delta t}$$

$$F_{1 \text{ ave}} = \frac{m(v_f - v_i)}{\Delta t_1}$$

Putting values

$$F_{1 \text{ ave}} = \frac{m(v_f - v_i)}{2 \times 10^{-3}} = \frac{(70)(-20)}{2 \times 10^{-3}} = \frac{-1400}{2 \times 10^{-3}}$$

$$F_{1 \text{ ave}} = -7 \times 10^5 \text{ N} \dots \dots \dots \text{Required Answer}$$

Solution (b): Now,

$$F_{2 \text{ ave}} = \frac{m(v_f - v_i)}{\Delta t_2}$$

$$F_{2 \text{ ave}} = \frac{70(0 - 20)}{45 \times 10^{-3}} = \frac{70(-20)}{45 \times 10^{-3}}$$

$$F_{2 \text{ ave}} = \frac{-1400}{45 \times 10^{-3}} = 3.1 \times 10^4 \text{ N}$$

The tree will act equal and opposite force on the passenger to stop him.

Therefore $F_{2 \text{ ave}} = 3.1 \times 10^4 \text{ N} \dots \dots \dots \text{Required Answer}$

Numerical Problem-3:

A 0.4 kg ball traveling with the speed of 15 ms^{-1} strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s, what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall?

Given Data:

Mass of the ball $= m = 0.4 \text{ kg}$

Speed of the ball = $v = 15 \text{ m/sec}$ Time interval = $\Delta t = 0.045 \text{ sec}$ **Required Data:**a) Momentum imparted to the wall = $\Delta P = ?$ b) Average force exerted on the wall = $F_{ave} = ?$ **Solution (a):** For elastic collision the change of momentum is:

$$\Delta P = 2mv$$

Putting values

$$\Delta P = 2(0.4)(15)$$

$$\Delta P = 12 \text{ kgm/sec} \dots \text{Required Answer}$$

Solution (b): As we know that

$$\Delta P = F_{ave} \Delta t$$

$$F_{ave} = \frac{\Delta P}{\Delta t}$$

Putting values

$$F_{ave} = \frac{12}{0.045} = 266.7 \text{ N}$$

$$F_{ave} = 266.7 \text{ N} \dots \text{Required Answer}$$

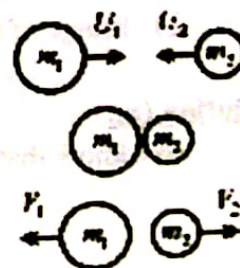
Numerical Problem-5:

One ball of mass 0.600 kg traveling 9.00 m/s to the right collides head on elastically with a second ball of mass 0.300 kg traveling 8.00 m/s to the left. After the collision, what are their velocities after collision?

(-2.33 m/s (2.33 m/s to right) and 14.67 m/s (14.76 m/s to left))

Given Data:Mass of the ball = $m_1 = 0.600 \text{ kg}$ Velocity of the ball before collision = $u_1 = 9 \text{ m/s}$ Mass of the 2nd ball = $m_2 = 0.300 \text{ kg}$ Velocity of the 2nd ball before collision = $u_2 = -8.00 \text{ m/s}$ **Required Data:**Velocity of the 1st ball after collision = $v_1 = ?$ Velocity of the 2nd ball after collision = $v_2 = ?$ **Solution:**

We know for elastic head on collision



$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2 \text{ ----- (i)}$$

Putting values in equation (i)

$$v_1 = \left(\frac{0.600 - 0.300}{0.600 + 0.300} \right) (9) + \left(\frac{2(0.300)}{0.600 + 0.300} \right) (-8.00)$$

$$v_1 = \left(\frac{0.3}{0.9} \right) (9) + \left(\frac{0.6}{0.9} \right) (-8.00)$$

$$v_1 = 3 - 5.3 = -2.3 \text{ m/s}$$

$$v_1 = -2.3 \text{ m/s}$$

$$V_1 = -2.3 \text{ m/s to left}$$

Negative sign show that the ball moves towards left side after collision

We also know that

$$V_2 = \left(\frac{2m_1}{m_1 + m_2} \right) u_1 - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_2$$

Putting values

$$V_2 = \left(\frac{2(0.600)}{0.600 + 0.300} \right) (9) - \left(\frac{0.600 - 0.300}{0.600 + 0.300} \right) (-8.00)$$

$$V_2 = 12 + 2.66$$

$$V_2 = 14.6 \text{ m/s}$$

$$V_2 = 14.6 \text{ m/s to right} \text{ Result}$$

Numerical Problem-6:

In a wedding a bullet is fired in air at a speed of 500 m/s making an angle of 60° with horizontal from an AK 47 rifle. (a) How high will the bullet rise? (b) What time would it take to reach ground? (c) How far would it go? (Ignore air resistance)

[(a) 9,560m (b) 88.3 s (c) 22,078 m]

Given Data:

Initial velocity of the bullet $= v_i = 500 \text{ m/s}$

Angle of elevation $= \theta = 60^\circ$

Required Data:

- Height of the bullet $= H = ?$
- Time of flight $= T = ?$
- Range of the bullet $= R = ?$

Solution (a):

We know that

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$

Putting values we get

$$H = \frac{(500)^2 (\sin 60)^2}{2(9.8)} \quad (g = 9.8 \text{ m/s}^2)$$

$$H = \frac{(250000)(0.8)^2}{19.6} = 9566 \text{ m}$$

$$H = 9566 \text{ m} \quad \text{..... Required Answer}$$

Solution (b):

As

$$T = \frac{2v_0 \sin \theta}{g} \quad \text{Putting values}$$

$$T = \frac{2(500) \sin 60}{9.8} = \frac{866.02}{9.8}$$

$$T = 88.3 \text{ sec} \quad \text{..... Required Answer}$$

Solution (c):

As

$$R = \frac{v_0^2 \sin^2 \theta}{g} \quad \text{Putting values}$$

$$R = \frac{(500)^2 \sin^2 (60)}{9.8}$$

$$R = \frac{(250000) \sin 120}{9.8}$$

$$R = \frac{(250000)(0.866)}{9.8}$$

$$R = 22092 \text{ m} \quad \text{..... Required Answer}$$

Numerical Problem-7:

The catapult hurls a stone of mass 32.0g with a velocity of 50.0 m/s at a 30.0° angle of elevation. (a) What is the maximum height reached by the stone? (b) What is its range?

(C) How long has the stone been in the air when it returns to its original height?

((a) 31.87 m (b) 5.1s (c) 220.8 m)

Given Data:

Mass of stone = $m = 32.0 \text{ g}$

Velocity of stone = $v_i = 50.0 \text{ m/s}$

Angle of elevation = $\theta = 30.0^\circ$

Required Data:

a) Maximum height of the stone = $H = ?$

b) Range of stone = $R = ?$

c) Time of flight = $T = ?$

Solution (a):

We know that

$$H = \frac{v_0^2 \sin^2 \theta}{2g} \quad \text{Putting values}$$

$$H = \frac{(50)^2 (\sin 30)^2}{2(9.8)} = \frac{(2500)(0.5)^2}{19.6}$$

$$H = \frac{625}{19.6} = 31.87m$$

$$H = 31.87m \quad \text{..... Required Answer}$$

Solution (b):

As

$$R = \frac{v_0^2 \sin^2 \theta}{g} \quad \text{Putting values}$$

$$R = \frac{(50)^2 \sin(2(30))}{9.8} = \frac{(2500) \sin 60^\circ}{9.8}$$

$$R = \frac{(2500)(0.866)}{9.8} = \frac{2165.06}{9.8}$$

$$R = 220.8m \quad \text{..... Required Answer}$$

Solution (c):

As

$$T = \frac{V_0 \sin \theta}{g} \quad \text{Putting values}$$

$$T = \frac{2(50) \sin 30}{9.8} = \frac{100(0.5)}{9.8}$$

$$T = 5.1 \text{sec} \quad \text{..... Required Result}$$

Unit # 4

WORK AND ENERGY

Comprehensive Questions (From Exercise)

Q. 1: Define work and show that it is the dot product of force and displacement. At what condition work done will be maximum or minimum?

Ans: Work

Physical Definition: When a force acts on a body and displaces it in its own direction, work is done on the body

Or

Mathematical Definition: The dot product of force and displacement is called work

Mathematically:

$$W = \vec{F} \cdot \vec{d}$$

Or

$$W = F d \cos\theta$$

Nature of work:

Work is a scalar quantity

Unit of work:

The unit of work is newton meter (N m) which is called Joule (J).

One Joule:

If a force of 1N displaces an object through one meter displacement, One Joule of work is done.

As

$$W = Fd \cos\theta$$

If

$$\theta = 0^\circ, \quad F = 1N, \quad d = 1m$$

Then

$$W = 1J$$

Dimensions:

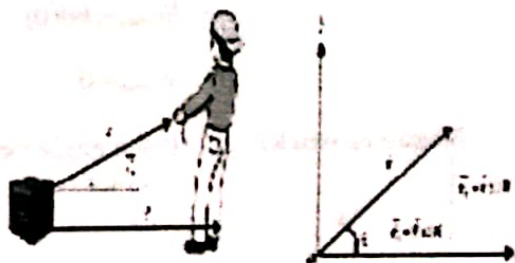
The dimensions of work according to its definition ($\vec{F} \cdot \vec{d}$) are $[ML^2T^{-2}]$.

To show that work is the dot product of force and displacement:

Consider a block is displaced by applying a force F as shown in the figure.

Work depends on two things force ($\vec{F}$) and displacement ($\vec{d}$).

One can see from the figure that the block has been displaced along x-axis and by the x component of force ($\vec{F}_x$), because the vertical



The only component of force that is in the direction of displacement is the horizontal component F_x .

component of force ($\vec{F}_y$) is in the upward direction, doing no work.

So $W = \vec{F}_x \cdot \vec{d}$

Or $W = (F \cos\theta) d$

Or $W = (\text{the component of } \vec{F} \parallel \text{to } d) d$

Using the definition of dot product of vectors.

$$W = \vec{F} \cdot \vec{d}$$

Hence work is the dot product of force ($\vec{F}$) and displacement ($\vec{d}$).

Conditions for maximum and minimum work:-

- i. Maximum work
- ii. Minimum work
- iii. Negative work

i) **Maximum work:** Work is maximum if $\theta = 0^\circ$, because $\cos 0^\circ = 1$; one (1) is the maximum value of $\cos\theta$.

$$W_{\max} = Fd \cos\theta$$

$$W_{\max} = Fd \cos 0^\circ$$

$$W_{\max} = Fd(1)$$

$$W_{\max} = Fd$$

ii) **Minimum work:** Work is minimum if $\theta = 90^\circ$, because $\cos 90^\circ = 0$; as explained below

$$W_{\min} = Fd \cos\theta$$

$$W_{\min} = Fd \cos 90^\circ$$

$$W_{\min} = Fd(0)$$

$$W_{\min} = 0$$

iii) **Negative work:** If the angle between the vectors is 180° i.e $\theta = 180^\circ$ and $\cos 180^\circ = -1$

As

$$W = \vec{F} \cdot \vec{d}$$

Or $W = Fd \cos\theta$

$$W = Fd \cos 180^\circ$$

$$W = Fd(-1)$$

$$W = -Fd$$

The work done by frictional force is always negative.

Q. 2: Define power and show that power is the dot product of force and velocity. What are the different units of power in our daily life?

Power: "The rate of doing work is called power".

Or

"The rate of transformation of energy is called power".

Mathematically:

Power is given as

$$P = \frac{W}{t}$$

Nature of Power:

Power is a scalar quantity

Dimensions of Power: According to the definition of power its dimensions are $[ML^2T^{-3}]$

To Show That: $P = \vec{F} \cdot \vec{V}$

as $P = \frac{W}{t}$ but $W = \vec{F} \cdot \vec{d}$

so $P = \frac{\vec{F} \cdot \vec{d}}{t}$

Or $P = \vec{F} \cdot \frac{\vec{d}}{t}$ where $\frac{\vec{d}}{t} = \vec{V}$

So $P = \vec{F} \cdot \vec{V}$

Hence power is the dot product of force ($\vec{F}$) and velocity ($\vec{V}$)

Average Power:

Total work done divided by total time is called average power

$$\langle P \rangle = \frac{\text{total work}}{\text{total time}}$$

Instantaneous Power:

The power calculated at a particular instant of time is called instantaneous power.

component of force ($\vec{F}_x$) is in the upward direction, doing no work.

So $W = \vec{F}_x \cdot \vec{d}$

Or $W = (F \cos\theta) d$

Or $W = (\text{the component of } \vec{F} \text{ || to } d) d$

Using the definition of dot product of vectors.

$$W = \vec{F} \cdot \vec{d}$$

Hence work is the dot product of force ($\vec{F}$) and displacement ($\vec{d}$).

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$$W_{\min} = 0$$

iii) **Negative work:** If the angle between the vectors is 180° i.e $\theta = 180^\circ$ and $\cos 180^\circ = -1$

As

$$W = \vec{F} \cdot \vec{d}$$

Or $W = Fd \cos \theta$

$$W = Fd \cos 180^\circ$$

$$W = Fd(-1)$$

$$W = -Fd$$

The work done by frictional force is always negative.

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Nature of Power:

Power is a scalar quantity

Dimensions of Power: According to the definition of power its dimensions are $[ML^2T^{-3}]$

To Show That: $P = \vec{F} \cdot \vec{V}$

as $P = \frac{W}{t}$ but $W = \vec{F} \cdot \vec{d}$

so $P = \frac{\vec{F} \cdot \vec{d}}{t}$

Or $P = \vec{F} \cdot \frac{\vec{d}}{t}$ where $\frac{\vec{d}}{t} = \vec{V}$

So

$$P = \vec{F} \cdot \vec{V}$$

Hence power is the dot product of force ($\vec{F}$) and velocity ($\vec{V}$)

Average Power:

Total work done divided by total time is called average power

$$\langle P \rangle = \frac{\text{total work}}{\text{total time}}$$

Instantaneous Power:

The power calculated at a particular instant of time is called instantaneous power.

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}$$

To calculate the exact instantaneous power the time interval Δt should be taken to be very small i.e. $\Delta t \rightarrow 0$

Different Units of Power Used in our Daily Life:

- i. Watt
- ii. Horse power (HP)
- iii. Ergs per second (erg/sec)
- iv. Foot-pound per minute.

i). Watt : It is the SI unit of power which is defined as,

"One joule of work done in one second is called one watt".

As $P = \frac{W}{t}$ so One watt = $\frac{\text{one Joule}}{\text{one second}}$

ii). Horse Power:-

746 joules of work is done in one second is called one Horse Power.

$$1 \text{ Hp} = 746 \text{ watts.}$$

Or

If the work is done at a rate of 550 ft-pound per second it is called one horse power.

Horse power is the unit of power in British engineering system.

iii). Erg per Second: 10^{-7} Joules of work done in one second is called Erg per Second.

iv). Foot-Pound per Minute: One foot-pound of work done in one minute is called one foot-pound minute.

Q. 3: Explain the work energy principle in the cases of the change in K.E of a body.

Ans: Work energy principle for K.E:

"The work done on a body is equal to the change in K.E of the body".

Mathematically:

Work done = change in K.E

$$W = \Delta K.E$$

$$W = K.E_f - K.E_i$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Derivation or Proof:

Consider a body in motion is acted on by a force $\vec{F}$ which changes its velocity from v_i to v_f .

The work done is given as $W = \vec{F} \cdot \vec{d}$ (i)

Where $\vec{F} = m\vec{a}$, $d = s = \frac{v_f^2 - v_i^2}{2a}$ (third equation of motion)

Put this in Eq (i) $W = m\vec{a} \left(\frac{v_f^2 - v_i^2}{2a} \right)$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W = K.E_f - K.E_i$$

Or $W = \Delta K.E$

Hence the work done on a body is equal to change in its K.E.

In Case of Frictional (resistive) forces:

If frictional or other dissipative forces are present, then some of the work is converted to K.E and some is utilized against these forces.

$$W = \Delta K.E + W_{\text{Friction}}$$

Or

$$W = \Delta K.E + W_{\text{resistive forces.}}$$

Q. 4: Prove that absolute P.E = $\frac{GM_e m}{R_e}$.

Ans: Absolute P.E: The work done on a body against gravitational field in taking the body from earth's surface to a point at infinity is called absolute potential energy.

Work done = absolute P.E

Due to varying values of "g" at different heights (altitudes) the equation $P.E = mgh$ cannot be used to calculate P.E. Hence absolute potential energy is needed to be calculated.

Proof or Derivation:

Consider a mass "m" is raised to a faraway point "N" which is at infinite distance r_N from earth's surface. The work done on this mass (object) is calculated by dividing the whole distance into small distances r_1, r_2, r_3 r_N as shown.

Work done along each path (distance) is calculated and finally all works done are added to get total work done which is the absolute P.E of the body.

i.e $W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + \dots + W_{N-1 \rightarrow N} \dots \dots \dots (i)$

As $\Delta W_{1 \rightarrow 2} = \langle F \rangle \cdot \Delta r$

Or $\Delta W_{1 \rightarrow 2} = \frac{GM_e m}{\langle r^2 \rangle} \Delta r$

Where $\Delta r = r_1 - r_2$

And $(r^2) = r_1 r_2$

$\Delta W_{1 \rightarrow 2} = \frac{GM_e m}{r_1 r_2} \cdot r_2 - r_1$

$\Delta W_{1 \rightarrow 2} = GM_e m \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$



The work done in moving the body of mass m from earth surface to very far off distance from earth where the force of gravity reduces.

Similarly work done from 2 to 3

i.e $W_{2 \rightarrow 3} = GM_e m \left(\frac{1}{r_2} - \frac{1}{r_3} \right)$

And $W_{N-1 \rightarrow N} = GM_e m \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right)$

Put all these works in equation (i)

$W = GM_e m \left(\frac{1}{r_1} - \frac{1}{r_2} \right) + GM_e m \left(\frac{1}{r_2} - \frac{1}{r_3} \right) \dots \dots \dots + GM_e m \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right)$

Take $GM_e m$ as common

$W = GM_e m \left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \dots + \frac{1}{r_{N-1}} - \frac{1}{r_N} \right)$

$W = GM_e m \left(\frac{1}{r_1} - \frac{1}{r_N} \right)$

As $r_N = \infty$ so $\frac{1}{r_N} = 0$

$W = \frac{GM_e m}{r_1}$

Or $W = P.E = \frac{GM_e m}{r}$

Since mass "m" is taken from earth's surface hence $r = R_e$

$P.E = \frac{GM_e m}{R_e}$ Which is the required proof.

Escape velocity:

"The minimum initial velocity which must be given to a body to get escaped from the influence of gravity is called escape velocity".

Proof or derivation: The K.E gained by the body to escape from gravity of earth is equal to the work done or absolute potential energy.

Hence from the law of conservation of energy

$$\text{K.E} = \text{absolute P.E}$$

$$\frac{1}{2} m v_{es}^2 = \frac{GM_e m}{R_e}$$



Gravitational field around the earth

Divide both sides by "m"

$$\frac{1}{2} v_{es}^2 = \frac{GM_e}{R_e}$$

Or $v_{es}^2 = 2 \frac{GM_e}{R_e}$

Or $v_{es} = \sqrt{2 \frac{GM_e}{R_e}}$ _____ (i)

But $g = \frac{GM_e}{R_e^2} \Rightarrow R_e^2 g = GM_e$

Put $GM_e = R_e^2 g$ in equation (i)

$$v_{es} = \sqrt{2 R_e^2 g} \text{ _____ (ii)}$$

As $R_e = 6.4 \times 10^6 \text{ m}, \quad g = 10 \text{ m/s}^2$
 $M_e = 6.0 \times 10^{24} \text{ Kg}, \quad G = 6.67 \times 10^{-11} \text{ N.m}^2 \text{ Kg}^{-2}$

Put these values either in (i) or (ii) we will get

$$v_{es} = 11.2 \text{ kms}^{-1}$$

The value of escape velocity is independent of mass of body (m) and just depends on the mass of gravitating (earth) body. Hence 11.2 kms^{-1} is the value of escape velocity for all bodies on earth.

Q. 6: Describe briefly various non-conventional sources of energy.

Ans: Non-Conventional Source of energy: The energy resources which can be renewed or recycled are called non-conventional sources of energy. These are also called as renewable sources of energy. Nature plays its role to renew these energies time and again.

- (i) Energy from bio mass.
- (ii) Energy from waves.
- (iii) Geothermal energy.
- (iv) Tidal energy.
- (v) Solar energy.
- (vi) Wind energy.

These non-conventional sources of energy are briefly discussed separately.

i) Energy from Biomass:

The energy generated from plants and animals is called biomass energy.

Raw materials for Biomass: The commonly used raw materials for energy from biomass are garbage, stuff lying on the ground, dead trees, tree branches, yard clippings, crop residues, wood chips, saw dust, dung cakes etc.

Mechanism of Biomass Energy Production: waste is brought to biomass power plant and fed into furnace for burning. The heat or steam is used to turn turbines and generators. Nowadays plastics are thought as target of high potential for biomass energy.

About 81 million tons/ annum biomass production has a huge potential to produce electricity.

ii) Energy from Waves: The energy passed by sea or ocean waves which are caused by motion of air (wind) is known as energy from waves.

Mechanism of Energy Production from Waves: Oscillating water column (owc) cause the specially designed devices to convert waves energy into electrical energy. The oscillating water column's submerged surface opens to the ocean below water surface. The waves through this structure cause. The water column to rise and fall with the wave which causes the air in the top structure to pressure and depressurize, this pressurization and depressurization of air turn the generator to produce electricity.

iii) Geothermal Energy:

The energy obtained from earth's inside heat is called geothermal energy

Causes of Geothermal Energy:

Springs, volcano and geysers are the main causes of geothermal energy

Mechanism of Generation of Geothermal Energy:

- Wells are drilled one to two miles deep into earth to pump hot water to surface.
- By reaching the surface, the pressure of hot water drops turns into steam which cause the turbine to spin.

- The steam is cooled and condensed and jumped back to make the process continuous.

iv) Tidal Energy:

The energy obtained from tides is called tidal energy. Tides are waves produced in ocean or sea due to gravitational pull of sun and moon on earth.

Causes of Tides: The gravitational pull of sun and moon causes water surface rise while earths pull and rotation attracts it down consequently give rise to tides.

Mechanism of Tidal Energy Generations:

Tidal energy is generated in tidal barrages by the use of specially designed devices, the rise and fall of tides causes the turbine to spin and generate electrical energy from tides.

Non-exercise Long Questions

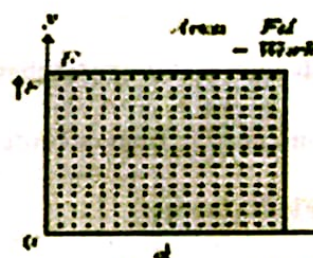
Q.1 Derive a formula for work done by variable force.

Ans: **Variable Force:**

"If the magnitude of a force is not constant (varying) is called variable force".

Examples:

- Spring Force i.e $F = kx$
- Gravitational force on a body moving upward.



Work done by a constant force.

If the force is variable it would have different values so, what value would one put in the equation $W = \bar{F} \cdot \bar{d}$? One cannot do so.

Formula for Work Done by Variable Force:

Let a block has been dragged over a bumpy and non-smooth surface as shown. If the block is moving upward or passing through a non-smooth surface, it would need larger force and vice versa. Since the force is variable so equation $W = \bar{F} \cdot \bar{d}$ cannot be directly used to calculate work.

To calculate the work done by variable force divide the path (distance) covered by block into small displacement elements, $\Delta d_1, \Delta d_2, \Delta d_3, \dots, \Delta d_n$. by doing so the force for each Δd becomes nearly constant.

Use $W = \bar{F} \cdot \bar{d}$ to calculate the work done for each Δd



, finally add up all works done to calculate total work done on the block.

$$W = \Delta w_1 + \Delta w_2 + \dots + \Delta w_n \quad (1)$$

$$\Delta w_1 = \vec{F}_{x1} \cdot \Delta \vec{d}_1$$

$$\Delta w_2 = \vec{F}_{x2} \cdot \Delta \vec{d}_2$$

$$\Delta w_n = \vec{F}_{xn} \cdot \Delta \vec{d}_n$$

Put these works in equation (1)

$$W = \vec{F}_{x1} \cdot \Delta \vec{d}_1 + \vec{F}_{x2} \cdot \Delta \vec{d}_2 + \dots + \vec{F}_{xn} \cdot \Delta \vec{d}_n$$

$$= F_{x1} \Delta d_1 \cos \theta_1 + F_{x2} \Delta d_2 \cos \theta_2 + \dots + F_{xn} \Delta d_n \cos \theta_n$$

$$W = \sum_{i=1}^n (F_{xi} \cos \theta) \Delta d_i$$

$$W = \lim_{\Delta d \rightarrow 0} \sum_{i=1}^n (F_{xi} \cos \theta) d_i$$

Δd is very small such that it approaches to Zero.

Q. 2: Define conservative field or force and show that gra

Conservative Field (force):

A field or force is said to be conservative if in a fi

i. work done is independent of paths

ii. work done along a closed path is zero

Examples: i. Electric Force ii. Spring Force

Examples of non-conservative force:

i. Frictional Force

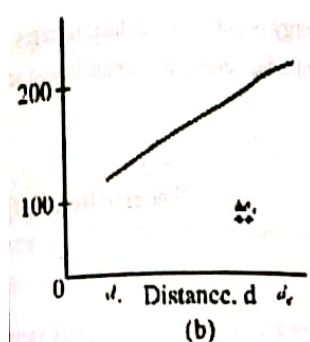
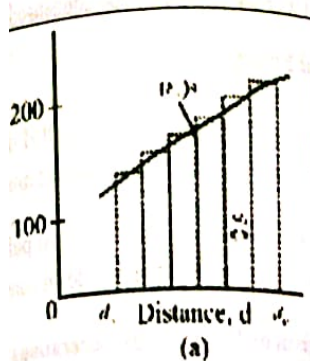
ii. Drag Force

iii. Tension in a string

iv. Normal Force

To show that gravitational field (force) is conservative:

Photon Series



Graphical illustration of work done by a varying force

gravitational field is conservative.

field;

iii. Gravitational Force

Let a body of a mass has been displaced along an elbow path as shown in figure.

Case # 1:

Let calculate work done along two different paths i.e

A → C and A → B → C

$$W_{A \rightarrow C} = \vec{F} \cdot \vec{d}$$

$$W_{A \rightarrow C} = mgd \cos \theta$$

From figure

$$d \cos \theta = d_1$$

So

$$W_{A \rightarrow C} = mgd_1 \quad \text{(i) Work done in two different paths}$$

Work done for A → B → C

$$W_{A \rightarrow B \rightarrow C} = W_{A \rightarrow B} + W_{B \rightarrow C}$$

$$W_{A \rightarrow B \rightarrow C} = mgd_1 \cos \theta + mgd_2 \cos \theta$$

$$W_{A \rightarrow B \rightarrow C} = mgd_1 \cos 0^\circ + mgd_2 \cos 90^\circ$$

$$W_{A \rightarrow B \rightarrow C} = mgd_1(1) + mgd_2 \cos(0)$$

$$W_{A \rightarrow B \rightarrow C} = mgd_1 + 0$$

$$W_{A \rightarrow B \rightarrow C} = mgd_1 \quad \text{(ii)}$$

Equations (i) and (ii) confirm that work done does not depend upon path. i.e work done is same (for different paths followed by block). Hence gravitational field (force) is conservative.

Case II:-

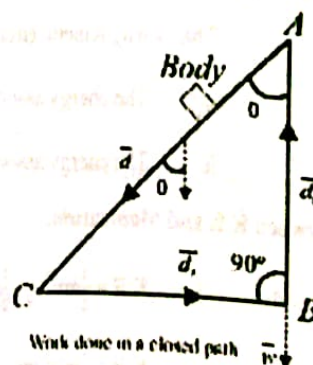
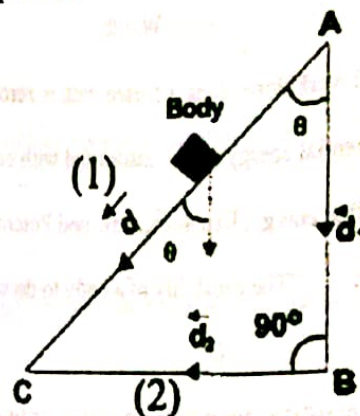
Let the block has been moved from C to B to A and again to point C along a closed path.

$$W = W_{C \rightarrow B} + W_{B \rightarrow A} + W_{A \rightarrow C}$$

$$W = mgd_2 \cos \theta + mgd_1 \cos \theta + mgd \cos \theta$$

From figure $d \cos \theta = d_1$

So



$$W = mgd_2 \cos 90^\circ + mgd_1 \cos 180^\circ + mg(d_1)$$

$$W = mgd_2(0) + mgd_1(-1) + mg(d_1)$$

$$W = 0 - mgd_1 + mgd_1$$

$$W = 0.$$

Hence the work done along a closed path is zero, thus gravitational field is conservative.

Note: Potential energy can be associated with conservative fields only.

Q. 3: Define energy, Explain Kinetic and Potential energy.

Energy: "The capability of a body to do work is called energy".

Or

"The agent which causes or tends cause a change in the state of a body is called energy.

Nature: Energy is a scalar quantity.

Dimensions:

The dimensions of Energy are $[ML^2T^{-2}]$ which are equal to the dimensions of work.

Units of Energy: The SI unit of energy is Joule (already defined). Other non-SI units of energy are Calorie, Electron-volt, Erg, Foot-Pound, Kilowatt-hour etc....

Kinetic Energy: "The energy of a body due to its motion is called Kinetic energy".

Mathematically:

$$K.E = \frac{1}{2}mv^2$$

Or
$$K.E = \frac{1}{2}m \vec{v} \cdot \vec{v}$$

This is why Kinetic energy is a scalar quantity.

Examples:

- i. The energy associated with a moving car.
- ii. The energy associated with a moving ball.

Relation between K.E and Momentum:

As
$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m \vec{v} \cdot \vec{v}$$

Or
$$K.E = \frac{1}{2}m^2v^2/m \quad \text{as } p = mv$$

$$K.E = \frac{1}{2}p^2/m$$

$$K.E = P^2 / 2m$$

Or

$$P = \sqrt{2mK.E}$$

This is the relation between kinetic energy and momentum.

Potential Energy:

"The energy of a body due to its position, state or configuration in a field is called Potential energy".

Gravitational Potential Energy:

"The energy of a body due to its position in the gravitational field is called gravitational potential energy".

Mathematically:

If a block of mass "m" is lifted to a height "h" work done is stored in the block in the form of gravitational potential energy.

$$P.E = W = \vec{F} \cdot \vec{d}$$

$$P.E = mgh \quad (\text{as } F=mg \text{ and } d=h)$$

Q. 4: Define and explain the law of conservation of energy.

Statement: "Energy can neither be created nor destroyed but can only be transformed from one form to another form"

Or

"The total energy of a system remains constant".

Mathematically:

$$\text{Total energy} = \text{constant}$$

Or

$$E = \text{constant}$$

Or

$$\Delta E = 0 \quad [\text{because if something is constant, change in it is zero}]$$

Conversion Or Transformation of Energy:

"The change of one form of energy into other form is called conversion or transformation of energy".

In a mathematical system

$$T.E = K.E + P.E$$

Thus K.E changes or transforms into P.E and vice versa.

Examples: If a hammer is raised high it gains P.E but on falling or striking with object it gains K.E that is

$$\text{Loss in P.E} = \text{Gain in K.E}$$

Q.5: Define and explain efficiency.

Efficiency:

"Efficiency of a machine is the ratio of output work to the input work".

Mathematically:

$$\text{Efficiency} = \eta = \frac{\text{output work}}{\text{input work}}$$

It is often expressed in terms of percentage called percentage efficiency;

$$\text{Percentage Efficiency} = \eta = \frac{\text{output work}}{\text{input work}} \times \frac{100}{100}$$

Explanation:

Efficiency is a pure dimensionless and unit less quantity.

Output work:

The work done by a machine is called output work.

Output work = load . Distance covered by load

$$\text{Output work} = F_{\text{out}} . D_{\text{out}}$$

Input Work:

The work done on a machine is called input work or input energy. The force applied on a machine is called effort.

Input work = Effort force . Effort distance

$$\text{Input work} = F_{\text{in}} . D_{\text{in}}$$

So

$$\eta = \frac{F_{\text{out}} . D_{\text{out}}}{F_{\text{in}} . D_{\text{in}}} \times \frac{100}{100}$$

Efficiency of a real machine:

The efficiency of a real machine is always less than 1 or 100% because some of the energy is dissipated by frictional or resistive forces in the form of heat, sound, light etc.

Examples:

- i. The efficiency of an incandescent lamp is 5%.
- ii. The efficiency of a petrol heat engine is 25-30%.
- iii. The efficiency of a diesel engine 34-40%.
- iv. The efficiency of a battery is 90%.

Efficiency of an Ideal Machine:

The efficiency of an ideal machine is 100% or 1.

i.e Output work = Input work

MCQ's

1. You push a heavy crate down a ramp at a constant velocity. Only four forces act on the crate. Which force does the greatest magnitude of work on the crate?
(A).The force of friction (B).The force of gravity
(C).The normal force (D).The force you are pushing
2. The force constant of a wire is 'k' and that of another wire is '3k'. When both the wires are stretched through some distance, if work done are W_1 and W_2 , then.....
(A). $W_2 = W_1$ (B). $W_2 = 9W_1$ (C). $W_1 = 3W_2$ (D). $W_2 = 3W_1$
3. Escape velocity on the surface of the earth is 11.2 km/s. If the mass of the earth increases to twice of its value and the radius of the earth becomes half, the escape velocity will be.....
(A).5.6 km/s (B).11.2 km/s (C).22.4 km/s (D).33.6 km/s
4. An example of non-conservative force is.....
(A).Electric force (B).Gravitational force (C).Frictional force (D).Magnetic force
5. When the speed of your car is doubled, by what factor does its kinetic energy increases?
(A). $\sqrt{2}$ (B).2 (C).4 (D).8
6. One horse power is equal to.....
(A).716 W (B).746 KW (C).746 MW (D).746 GW
7. Work is said to be negative when F and d are.....
(A).Parallel (B).Anti-parallel (C). Perpendicular (D).At 45°
8. Two bodies of masses m_1 and m_2 have equal momentum, their kinetic energies E_1 and E_2 are in the ratio.....
(A). $\sqrt{m_1} : \sqrt{m_2}$ (B). $m_1 : m_2$ (C). $m_2 : m_1$ (D). $\sqrt{m_1^2} : \sqrt{m_2^2}$
9. The atmosphere is held to the earth by
(A).Winds (B). Gravity (C).Clouds (D).The rotation of Earth
10. If momentum is increased by 20%, then kinetic energy increases by.....
(A).44% (B).55% (C).66% (D).77%

11. Is K.E of a body becomes four times of the initial value, then new momentum will
 (A).Become twice of its initial value (B).Become three times of its initial value
 (C).Become four times of its initial value (D).Remain constant
12. Two bodies with kinetic energies in the ratio 4:1 are moving with equal linear momentum. The ratio of their masses is
 (A).1 : 2 (B).1 : 1 (C).4 : 1 (D).1 : 4
13. A body of mass 5 kg is moving with a momentum of 10 kg m/s. A force of 0.2 N acts on it in the direction of motion of the body for 10 s. The increase in its kinetic energy is
 (A).2.8 J (B).3.2 J (C).3.8 J (D).4.4 J

If force and displacement of a particle in the direction of force are doubled, work would become

- (A).Double (B).4 times (C).Half (D).1/4 times

Answer Keys:

1	A	5	C	9	B	13	D
2	D	6	A	10	A	14	B
3	C	7	B	11	A		
4	C	8	C	12	D		

MCQ's solution

1. Displacement covered only due to the force you pushing, therefore only this force will did work.

Ans: (d) the force you pushing

2. Work done on stretching the string of constant "K" through displacement "x" is given by

$$w_1 = \frac{1}{2} k x^2 \dots\dots\dots(i)$$

While the work done on stretching the string of constant "3k" through the same displacement "x" is given by

$$w_2 = 3 \left(\frac{1}{2} k x^2 \right) \dots\dots\dots(ii)$$

By equation (i) and (ii) we get

$$w_2 = 3w_1$$

Ans: (d) $w_2 = 3w_1$

3. As we know that

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} \dots\dots\dots(i)$$

When $M_E' = 2M_E$ and $R_E' = 2R_E$

$$v'_{esc} = \sqrt{\frac{2G \times 2M_E}{\frac{R_E}{2}}} = \sqrt{4 \left(\frac{2GM_E}{R_E} \right)} = 2 \sqrt{\left(\frac{2GM_E}{R_E} \right)} = 2v_{esc}$$

$$v'_{esc} = 2(11.2) = 22.4 \text{ km/sec}$$

Ans: (C) 22.4 km/sec

4. Ans: (C) Frictional Force

5. As we know that

$$K.E = \frac{1}{2}mv^2 \dots\dots\dots(i)$$

When $v' = 2v$ then equation (i) becomes

$$K.E' = \frac{1}{2}mv'^2$$

$$K.E' = \frac{1}{2}m(2v)^2$$

$$K.E' = 4 \frac{1}{2}m(v)^2$$

Ans: (C) $K.E' = 4 K.E$

6. Ans: (a) 746 watt

7. We know that

$$W = \vec{F} \cdot \vec{s}$$

$$W = FS \cos \theta \quad (\text{if } \theta = 180^\circ)$$

$$W = FS \cos(180^\circ)$$

$$W = Fs(-1)$$

$$W = -Fs$$

Ans: (b) Anti parallel

So work done is said to be negative when $\vec{F}$ and $\vec{d}$ are Anti parallel.

8. Momentum of first body of mass " m_1 " is

$$P_1 = \sqrt{2m_1 E_1} = \sqrt{2m_2 E_2}$$

$$P_1 = P_2 = \sqrt{2m_1 E_1} = \sqrt{2m_2 E_2}$$

Squaring both sides

$$(\sqrt{2m_1 E_1})^2 = (\sqrt{2m_2 E_2})^2$$

$$2m_1 E_1 = 2m_2 E_2$$

$$m_1 E_1 = m_2 E_2$$

$$\frac{E_1}{E_2} = \frac{m_2}{m_1} = m_2 = m_1$$

dividing both sides by '2' we get

Ans: (c) $m_1 : m_2$

9. Ans: (b) Gravity

10.

$$\text{Let K.E} = \frac{1}{2}mv^2$$

$$\text{K.E} = \frac{1}{2} \frac{m^2}{m} v^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{1}{2} \frac{P^2}{m} = \frac{P^2}{2m}$$

If momentum is increased by 20% then

$$\text{K.E}_f = \left(\frac{P^2 + 20\%P^2}{2m} \right) = \left(\frac{P^2 + \frac{20}{100}P^2}{2m} \right) = \frac{P^2(1.2)^2}{2m} = (1.44) \frac{P^2}{2m} = (1.44) \text{ K.E}$$

Now

$$\Delta \text{K.E} = \text{K.E}_f - \text{K.E}_i = (1.44)\text{K.E} - \text{K.E} = (0.44)\text{K.E}$$

$$\text{Percent increase will be } \frac{\Delta \text{K.E}}{\text{K.E}} \times 100 = \frac{(0.44)\cancel{\text{K.E}}}{(\cancel{\text{K.E}})} \times 100 = 44\%$$

Ans: (a) 44%

11. We know that

$$P_1 = \sqrt{2mK.E_1} \dots\dots\dots(i)$$

When K.E becomes four times i.e $K.E_2 = 4 K.E_1$ then equation (i) becomes

$$P_2 = \sqrt{2mK.E_2} = \sqrt{2m(4K.E_1)} = \sqrt{4(2mK.E_1)} = \sqrt{4} \sqrt{2mK.E_1}$$

$$P_2 = 2\sqrt{2mK.E_1} \dots\dots\dots(ii)$$

Comparing equation (i) and (ii) we get

$$P_2 = 2P_1$$

Ans: (a) becomes twice its initial value

12. According to the given equation we have

$$\text{K.E}_1 : \text{K.E}_2 = 4 : 1$$

$$\frac{K.E_1}{K.E_2} = \frac{4}{1} \dots\dots\dots(i)$$

Now

$$K.E_1 = \frac{P_1^2}{2m_1} \quad \text{and} \quad K.E_2 = \frac{P_2^2}{2m_2}$$

So equation (i) becomes $\frac{\frac{P_1^2}{2m_1}}{\frac{P_2^2}{2m_2}} = \frac{4}{1} = \frac{P_1^2}{P_2^2} \times \frac{2m_2}{2m_1} = \frac{4}{1} = \frac{P_1^2}{P_2^2} = \frac{4m_1}{m_2}$

$$\frac{P_1}{P_2} = 2\sqrt{\frac{m_1}{m_2}} \quad \text{we are given that } P_1 = P_2 = P_3 \dots\dots \text{ so we get}$$

$$\frac{P}{P} = 2\sqrt{\frac{m_1}{m_2}}$$

$$1 = 2\sqrt{\frac{m_1}{m_2}}$$

$$\frac{1}{2} = \sqrt{\frac{m_1}{m_2}}$$

$$\frac{1}{4} = \frac{m_1}{m_2}$$

$$\frac{m_1}{m_2} = \frac{1}{4} = m_1:m_2 = 1:4$$

Ans: (d) 1:4

13. We have given that

$$m = 5\text{kg}, P_i = 10 \text{ kg m/sec}, F = 0.2 \text{ N}, \Delta t = 10\text{sec}$$

The change in K.E is given by

$$\frac{1}{2(5)}[(12)^2 - (10)^2] = \frac{1}{10}[144 - 100] = \frac{44}{10} = 4.4\text{J}$$

$$\Delta K.E = 4.4\text{J}$$

$$\Delta K.E = \frac{P_f^2}{2m} - \frac{P_i^2}{2m} \Rightarrow \frac{1}{2m}[P_f^2 - P_i^2] \dots\dots\dots(i)$$

As we know that

$$F = \frac{\Delta P}{\Delta t} \Rightarrow \Delta P = F \Delta t$$

$$\Rightarrow \Delta P_f - P_i = F \Delta t$$

$$\Rightarrow P_f = F \Delta t - P_i \quad \dots\dots\dots(ii)$$

Putting values in equation (ii) we get

$$P_f = (0.2)(10) + 10 = 12$$

$$P_f = 12 \text{ kg m/sec}$$

Putting values in equation (i) we get

$$\Delta K.E = W' = 4W$$

$$\Delta K.E = 4.4 \text{ J}$$

Ans: (d) 4.4 J

14. We know that

$$W = F \cdot S \quad \dots\dots\dots(i)$$

$$\text{When } F' = 2F \text{ and } S' = 2S$$

Then equation (i) becomes

$$W' = F'S'$$

$$W' = (2F)(2S)$$

$$W' = 4FS \quad \dots\dots\dots(ii)$$

Putting equation (i) and equation (ii), we get

$$W' = 4W$$

Ans: (b) 4 Times

Short Questions

Q. 1: If a bucket is taken to the bottom of a well, does the bucket possess any P.E? Explain.

Explanation: A bucket is taken to the bottom of a well, we need to do work against the up-thrust ($\vec{F}_h$) of water.

- This work done is stored in the form of P.E in the bucket

$$P.E_B = W \quad (i)$$

- Since the work done is given by

$$W = \vec{F} \cdot \vec{d}$$

$$W = \vec{F} d \cos 0^\circ$$

- Since the $\vec{F}_h$ (up-thrust) is acting upward on the bucket and bucket is moving downward i.e. the $\vec{F}_h$ and displacement are in opposite direction. The angle between $\vec{F}_h$ and displacement is 180° .

$$\theta = 180^\circ$$

Put in equation (i)

$$P.E = W = \vec{F}_h d \cos(180^\circ) \quad (\cos(180^\circ) = -1)$$

$$P.E = W = \vec{F}_h d (-1)$$

$$P.E = W = - \vec{F}_h d$$

$$P.E = -(\vec{F}_h d)$$

$$P.E = -W$$

Conclusion: The work done is negative and therefore the P.E stored in the bucket is also negative.

Q. 2: when an arrow is shot from its bow, it has K.E, from where does it get K.E?

Ans: Origin of the K.E of arrow:

When an arrow is shot from its bow it has K.E, Which comes from the elastic P.E of the string of the bow.

Explanation:

Stretching the string of the bow:

- Before shooting an arrow, the string of the bow must be stretched.
- To stretch the string of the bow, Work (W) is done on it. Which is stored in the string in the form of elastic P.E

Energy transfer:

When the arrow is shot, the stretched string is released and the stored elastic P.E in it is transferred to the arrow in the form of its K.E.

$$(K.E)_{\text{Arrow}} = (P.E)_{\text{Bow}}$$

Conclusion:

Thus when an arrow is shot from a bow, its K.E comes from the P.E in the string of the bow.

Q. 3: Does a hydrogen filled balloon possess any P.E? Explain.

Ans: Thesis Statement:

Yes, a hydrogen filled balloon possesses P.E.

Explanation:

- The density of a hydrogen filled balloon is very much less than the air (mixture of gases).
- The weight of this hydrogen filled balloon is much smaller than the upthrust of surrounding air on it.

$$W_B < \overline{F_{th}}$$

$$M_{HG} < \overline{F_{th}}$$

W_B = Weight of balloon

$\overline{F_{th}}$ = upthrust of the air

- Since upthrust of the air on the balloon is greater, therefore the thrust will move the balloon in upward direction.
- The balloon has the ability to do work due to its P.E

Conclusion:

A hydrogen filled balloon possesses a negative gravitational P.E due to the upthrust of air on it.

Q. 4: Is K.E a vector quantity?

Ans: Thesis Statement:

K.E is not a vector quantity, it is a scalar quantity.

Explanation:

The K.E is defined as $K.E = \frac{1}{2}mv^2$

m = mass of the object (Scalar)

v^2 = the magnitude of velocity (Scalar).

Square of velocity v^2 :

Since velocity is a vector quantity but the dot product of $\vec{v}$ and $\vec{v}$ result in a scalar quantity.

As $\vec{v} \cdot \vec{v} = v v \cos\theta$

$$\vec{v} \cdot \vec{v} = v^2 \cos(0^\circ)$$

$$\vec{v} \cdot \vec{v} = v^2$$

Since V means the magnitude of " v " and magnitude is always scalar.

Conclusion:

Thus the K.E is a scalar quantity as energy can never be a vector.

Q. 5: What happens to the K.E of a bullet when it penetrates into a target?

Ans: Thesis Statement:

The K.E of a bullet is transferred to the target and is converted into other forms of energy.

Explanation:

A bullet moving with a velocity possesses (K.E_B).

- When the bullet penetrates into the target, it's K.E is converted to the following forms of energy.
- Some of the K.E is used to overcome the resistance offered to bullet by the target material.
- A part of K.E appears in the form of sound produced during the collision of bullet with the target.
- Some of the energy is used in the heating of the target material.
- A small portion is converted to light during the collision (Spark).

Conclusion:

The K.E of a bullet appears in the form of heat energy, sound energy and light energy and also the K.E of particles of the target.

Q. 6: Does the tension in the string of a swinging pendulum do any work? Explain.

Ans: Thesis Statement:

Work is done by the tension in the string of a swinging pendulum.

Explanation:

The tension in the string of a swinging pendulum is always acting along the string of the pendulum.

$$\text{i.e. } T \perp \vec{d} \quad \theta = 90^\circ$$

Since work done is given by

$$W = \vec{F} \cdot \vec{d} \quad \text{as } F = T$$

$$W_T = \vec{T} \cdot \vec{d}$$

$$W_T = Td \cos \theta \quad \text{as } \theta = 90^\circ$$

$$W_T = Td(0) \quad W_T = 0.$$

Conclusion:

Thus no work is done by the tension in the string of a swinging pendulum.

Q. 7: A meteor when enters into the earth's atmosphere burns. What happens to its energy?

Ans: Thesis Statement:

A meteor when enters into the earth's atmosphere burns and its energy is converted into other forms of energy.

Explanation:

Upon entering into the earth's atmosphere a meteor burns due to friction with the air.

Conservation of momentum and energy says that the meteor and the atmosphere form a closed system, where both the momentum and energy are conserved.

Conversion of energy:

During the burning of meteor its energy is converted into heat energy, light energy, sound energy and some of the energy is transferred to the atmosphere's particles.

Conclusion:

The energy of a burning meteor is converted into heat, light, sound and the K.E of air particles.

Q. 8: What type of energy is stored in the spring of a watch?

Ans: Thesis Statement:

Elastic P.E is stored in the spring of a watch.

Mathematically: $P.E = \frac{1}{2} Kx^2$

Where x = extension in the spring

And k = spring constant

Explanation:

When the spring of a watch is wound, the spring stretches and work is done on the spring.

This work done is stored in spring in the form of elastic P.E.

Conclusion:

Thus the energy stored in the spring of a watch is the elastic P.E.

Q. 9: A man drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

Ans: Thesis Statement:

When a man drops a cup from certain height, which breaks into pieces, the following energy changes are involved.

Energy changes involved:-

- Initially the cup has P.E at height "h".
- When it is released its P.E is converted into K.E.
- On striking the ground the cup breaks into pieces and its energy is converted into,
 - Heat energy
 - Sound energy
 - K.E of the pieces of the cup.

Conclusion:

When a cup breaks into pieces, energy changes involved are.

Heat, light and sound energies.

Q. 10: A man rowing boat upstream is at rest with respect to shore, is he doing work?

Ans: Thesis Statement:

No, a man rowing boat upstream at rest with respect to shore, is doing zero (no) work. Because the man is rowing but he is at rest and no displacement is covered.

Explanation:

The work done by a body is defined as,

$$W = \vec{F} \cdot \vec{S}$$

Or

$$W = FS \cos \theta$$

In which

$F = \text{force}$ and $S = \text{displacement}$

If

$S = 0$ put in above equation

⇒

$$W = F(0) \cos \theta$$

⇒

$$W = 0$$

This proves that the work done is zero.

Conclusion:

It is thus concluded that if a man rowing boat upstream is at rest, he is doing no work.

Q. 11: Why energy savers are used instead of normal bulbs.

Ans: Thesis Statement:

Energy savers are used instead of normal bulbs due to the following reasons:

1. Lower consumption of energy
2. Longer life span
3. Higher Intensity of light
4. Safer than bulbs
5. Higher efficiency

Explanation:

- Energy savers consume less amount of electrical energy as compared to ordinary bulbs
- Energy savers are durable and have a longer life as compared to normal bulbs.
- Energy savers produce more intense light which cannot be produced by normal light bulbs.
- Energy savers produce very little amount of heat therefore they are more safer than normal bulbs
- The efficiency of energy savers is far more higher than normal light bulbs.

Conclusion:

Thus it is concluded that for the above mentioned reasons, energy savers are used instead of normal bulbs.

Assignments

Assignment. 4.1: What is the power of an airplane of mass 3000 kg if on the runway it is capable of reaching a speed of 80 m/s from rest in 4.0 s?

Given data:

Mass of airplane = $m = 3000 \text{ kg}$

Speed of airplane = $v = 80 \text{ m/sec}$

Time taken = $t = 4 \text{ sec}$

Required:

Power of airplane = $P = ?$

Solution:

As we know that

$$P = \frac{W}{t} = \frac{F \cdot S}{t} = F \frac{S}{t} = F v$$

As

$$F = \frac{mv_f - mv_i}{\Delta t}$$

Or

$$P = \left(\frac{mv_f - mv_i}{\Delta t} \right) \cdot v \quad \text{Put values}$$

$$\Rightarrow P = \frac{3000 \times 80 - 3000 \times 0}{4}$$

Result:

$$\Rightarrow P = 4.8 \times 10^6 \text{ watt}$$

Assignment. 4.2: How fast the moon need to travel in order to escape the gravitational pull of earth, if earth has a mass of $5.98 \times 10^{24} \text{ kg}$ and the distance from earth to moon is $3.84 \times 10^8 \text{ m}$?

Given data:

Mass of earth = M_E

Distance between earth and moon = $r = 3.84 \times 10^8 \text{ m}$

Radius of earth = $R_E = 6.4 \times 10^6 \text{ m}$

Gravitational constant = $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

Required:

Escape velocity = $V_{\text{esc}} = ?$

Solution:

As we know that

$$V_{\text{esc}} = \sqrt{\frac{2GM_E}{R}}$$

Putting values

$$V_{\text{esc}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{3.84 \times 10^8}}$$

Result:

$$V_{\text{esc}} = 1441 \text{ m/s}$$

bottom of the hill is 10ms^{-1} , because of friction. How much work is done by friction?

(a. 31.3 ms^{-1} , b. -8840 J)

Given data:

Height of hill = $h = 100\text{ m}$

Angle of inclination = $\theta = 30^\circ$

Mass of person = $m = 20\text{ kg}$

Required:

(a) Speed of person at the bottom = $v = ?$

(b) Work done by friction for $W_f = ?$

Solution: (a)

We resolve the weight of the person then the component " $w \cos\theta$ " balances the normal force while the component " $w \sin\theta$ " is responsible for the downward motion of the person.

Now P.E at the top is given by,

$$\text{P.E}_{\text{top}} = w.h = (w \sin\theta) h$$

$$\text{P.E}_{\text{top}} = mgh \sin\theta$$

$$\text{P.E}_{\text{top}} = (20)(9.8)(100) \sin 30^\circ$$

$$\text{P.E}_{\text{top}} = 9800 \text{ Joule.}$$

Now K.E at the top is given by,

$$\text{K.E}_{\text{top}} = \frac{1}{2}mv^2$$

$$\text{K.E}_{\text{top}} = \frac{1}{2}(20)(2) = 40 \text{ J}$$

$$\text{K.E}_{\text{top}} = 40 \text{ Joule}$$

The total energy at the top is given by,

$$(E_{\text{total}})_{\text{top}} = \text{P.E}_{\text{top}} + \text{K.E}_{\text{top}}$$

$$(E_{\text{total}})_{\text{top}} = (9800 + 40) \text{ Joule}$$

$$(E_{\text{total}})_{\text{top}} = 9840 \text{ J} \quad \dots\dots\dots(i)$$

Now P.E at the bottom is

$$(P.E)_{\text{bottom}} = 0 \quad [\text{because height becomes zero}]$$

And the K.E at the bottom is given by,

$$(K.E)_{\text{bottom}} = \frac{1}{2}mv^2$$

Now total energy at bottom is given by,

$$(E_{\text{total}})_{\text{bottom}} = (P.E)_{\text{bottom}} + (K.E)_{\text{bottom}}$$

$$(E_{\text{total}})_{\text{bottom}} = 0 + \frac{1}{2}mv^2 \quad \dots\dots\dots(ii)$$

Now according to law of conservation of energy we have,

$$(E_{\text{total}})_{\text{top}} = (E_{\text{total}})_{\text{bottom}}$$

$$9840 = \frac{1}{2}mv^2$$

$$v^2 = \frac{2(9840)}{m} = \frac{2 \times 9840}{20}$$

$$v^2 = 984$$

$$\sqrt{v^2} = \sqrt{984}$$

Result: $v = 31.3 \text{ m/sec}$ [Required velocity of person at the bottom]

Solution: (b)

When $v = 10 \text{ m/sec}$ and air resistance is present, then we have

Total energy at top + work done against friction = K.E at bottom

$$9840 \text{ J} + \text{work done} = \frac{1}{2}mv^2$$

$$\text{Work done} = \frac{1}{2}mv^2 - 9840$$

$$\text{Work done} = \frac{1}{2}(20)(10)^2 - 9840$$

$$\text{Work done} = 1000 - 9840$$

Result: $\text{work done} = -8840 \text{ J}$

Numerical Problems

Problem No1: A 70 kg man runs up a long flight of stairs in 4s. The vertical height of the stair is 4.5m. Calculate his power.

(P 7.7×10^2 Watts)**Given data**Mass of the man = $m = 70 \text{ kg}$ Time interval = $t = 4 \text{ sec}$ Height of stairs = $h = 4.5 \text{ m}$ **Required:**Power = $p = ?$ **Solution:**

As we know that

$$P = \frac{\text{work}}{\text{time}} = \frac{w}{t}$$

$$P = \frac{P.E}{t} = \frac{mgh}{t}$$

Putting values

$$\text{Answer : } P = \frac{(70\text{kg})(9.8\text{m/s}^2)(4.5\text{s})}{4} = 771.75 \text{ watt} = 7.7 \times 10^2 \text{ Watts}$$

Problem No2: A body of mass 2.0 Kg is dropped from a rest position 5m above the ground. What is its velocity at height of 3.0 m above the ground? ($v = 6.3 \text{ m s}^{-1}$)

Given data:Mass of the body = $m = 2 \text{ kg}$ Total height = $h = 5 \text{ m}$ Height where velocity to be found = $h_2 = 3\text{m}$ **Required:**Velocity at height of $3\text{m} = v = ?$

When the body reaches to point "B" and we ignore the air resistance then at point "B" we have,

Loss in P.E = Gain in K.E

$$mgh_1 = \frac{1}{2}mv^2$$

$$gh_1 = \frac{1}{2}v^2$$

$$v^2 = 2gh_1$$

$$v = \sqrt{2gh_1}$$

Putting values

$$v = \sqrt{2(9.8)(2)}$$

$$v = 6.26 = 6.3 \text{ m/sec}$$

Answer

$$v = 6.3 \text{ m/sec}$$

Problem No3: A man pulls a trolley through a distance of 50m by applying a force of 100N which makes an angle of 30° with horizontal. Calculate the work done by the man.

(4330 J)

Given data:Distance travelled = $s = 50 \text{ m}$ Applied force = $f = 100 \text{ N}$ Angle = $\theta = 30^\circ$ **Required:**Work done = $W = ?$ **Solution:**

As we know that

$$W = \vec{F} \cdot \vec{s}$$

$$W = FS \cos \theta \quad \text{--- (i)}$$

Putting values in equation(i) we get

$$W = 100 \times 50 \times \cos 30^\circ$$

$$W = 5000 \times 0.866$$

Result: $W = 4330 \text{ J}$

Problem. 4: The roller-coaster car starts its journey from a vertical height of 40m on the first hill and reaches a vertical height of only 25m on the second hill, where it slows to a momentary stop. It traveled a total distance of 400 m. Determine the thermal energy produced and estimate the average friction force on the car whose mass is 1000 kg. **(370N)**

Given dataHeight of 1st hill = $h_1 = 40 \text{ m}$ Height of 2nd hill = $h_2 = 25 \text{ m}$ Mass of car = $m = 1000 \text{ Kg}$ Distance travelled = $d = 400 \text{ m}$ **Required:**(a) Thermal energy = $E_{\text{thermal}} = ?$ (b) Average frictional force = $F_{\text{force}} = ?$ **Solution: (a)**Potential energy of car on top of 1st hill is given by

$$P.E_1 = mgh_1$$

putting values

$$P.E_1 = 1000 \times 9.8 \times 40$$

$$P.E_1 = 39200 \text{ J}$$

The P.E at the top of 2nd hill is given by

$$P.E_2 = mgh_2$$

putting values

$$P.E_2 = 1000 \times 9.8 \times 25$$

$$P.E_2 = 245000 \text{ J}$$

When the coaster car moves from 1st hill to 2nd hill the inter conversion of K.E and P.E takes place, so we have

$$\Delta K.E = \Delta P.E$$

$$\Delta K.E = P.E_2 - P.E_1$$

$$\Delta K.E = 245000 - 392000$$

$$\Delta K.E = -147000 \text{ J}$$

The thermal energy will be given by

$$E_{th} = -\Delta K.E$$

$$E_{th} = -(-147000)$$

Result:

$$E_{th} = 147000 \text{ J} = 147 \text{ KJ}$$

Solution: (b)

$$\text{Work} = \Delta K.E$$

$$\text{Work} = E_{th}$$

$$F_f d = 147000$$

$$F_f = \frac{147000}{d} = \frac{147000}{400}$$

Answer:

$$F_f = 368 \text{ N}$$

Problem. 5: A man whose mass is 70kg walks up to the third floor of a building which is 12m above the ground in 20s. Find his power in watts and hp.

$$(P \text{ 411.6 watt} = 0.55 \text{ hp})$$

Given data:

$$\text{Mass of man} = m = 70 \text{ Kg}$$

$$\text{Height of the 3rd floor} = h = 12 \text{ m}$$

$$\text{Time taken} = t = 20 \text{ sec}$$

Required:

$$\text{Power} = ?$$

Solution:

As we know that

$$\text{Power} = \frac{\text{work}}{\text{time}} \dots\dots\dots(i)$$

As the man walks up to the 3rd floor, the work done by him stored in the form of potential energy.

$$\text{i.e. work} = P.E$$

therefore equation (i) becomes as

$$P = \frac{P.E}{t} = \frac{mgh}{t}$$

Putting values

$$P = \frac{(70)(9.8)(12)}{20}$$

Result:

$$P = 411.6 \text{ Watt}$$

To convert watt in "hp" we know that;

$$1 \text{ hp} = 746 \text{ watt}$$

$$1 \text{ watt} = \frac{1}{746} \text{ hp}$$

$$\text{so } 411.6 \text{ watt} = \frac{411.6}{746} \text{ hp} = 0.55 \text{ hp}$$

Result:

$$P = 411.6 \text{ watt} = 0.55 \text{ hp}$$

Problem. 6: To what height can a 400W engine lift a 100 kg mass in 3s? ($h = 0.122\text{m}$)

Given data:

$$\text{power of engine} = P = 400 \text{ w}$$

$$\text{Mass} = m = 100 \text{ kg}$$

Time taken = $t = 3 \text{ sec}$

Required:

Height = ?

Solution:

As we know that

$$P = \frac{W}{t} = \frac{P.E}{t} = \frac{mgh}{t}$$

$$P = \frac{mgh}{t} \Rightarrow h = \frac{Pt}{mg} \dots\dots\dots(i)$$

Putting values in equation (i) we get

$$h = \frac{(400)(3)}{(100)(9.8)} = 1.22 \text{ m}$$

Result: $h = 1.22 \text{ m}$

Problem. 7: A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1} . If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction. **(31.9m, 6.7J, 0.3N)**

Given data:

Mass of ball = 100 gm = 0.1 kg

Speed of ball = $v = 25 \text{ m/sec}$

Required:

- a. Height = ?
- b. If height is $h = 25 \text{ m}$ then find
 - i. Work done against friction $W_f = ?$
 - ii. Force of friction = $f = ?$

Solution: (a)

If no energy is lost, then

$$(K.E)_{\text{loss}} = (P.E)_{\text{gain}}$$

$$\frac{1}{2} m v^2 = mgh$$

$$h = \frac{v^2}{2g}$$

Putting values

$$h = \frac{25^2}{2(9.8)} = 31.9 \text{ m}$$

Result: $h = 31.9 \text{ m}$

Solution: (b)

- (i). Now if $h' = 25 \text{ m}$ and friction is there then loss in K.E = gain in W_f against friction

$$W_f = \frac{1}{2}(0.1)(25)^2 - (0.1)(9.8)(25) = 6.7 \text{ J}$$

Result: $W_f = 6.7 \text{ J}$

(ii). Now to find friction force we know that

$$W_f = fh$$

$$f = \frac{W_f}{h}$$

Putting values

$$f = \frac{6.67}{25} = 0.3 \text{ N}$$

Result: $f = 0.3 \text{ N}$

Problem. 8: An object of mass 1000 g falls from a height of 30m on the sand below. If it penetrates 4cm into the sand, what opposing force is exerted on it by the sand? Neglect air friction. ($f = 7350 \text{ N}$)

Given data:

Mass of an object = $m = 1000\text{g} = 1 \text{ kg}$

Height = $h = 30 \text{ m}$

Penetration in sand = $d = 4 \text{ cm} = 0.04 \text{ m}$

Required:

Opposing force = $f = ?$

Solution:

If we neglect air friction then by work energy theorem

$$(P.E)_{\text{loss}} = (K.E)_{\text{gain}} \quad \therefore \text{As } P.E = mgh$$

$$(K.E)_{\text{gain}} = mgh$$

Putting values

$$(K.E)_{\text{gain}} = (1)(9.8)(30) = 294 \text{ J}$$

$$(K.E)_{\text{gain}} = 294 \text{ Joule}$$

When an object touches the surface of the sand and penetrates into it, then the $(K.E)_{\text{gain}}$ will lost in the work done against friction.

$$(K.E)_{\text{gain}} = W_f$$

$$W_f = (K.E)_{\text{gain}}$$

$$\therefore W_f = fd$$

$$fd = 294 \text{ J}$$

$$f = \frac{294}{d} = \frac{294}{0.04} = 7350 \text{ N}$$

Result: $f = 7350 \text{ N}$

Problem. 9: A body of mass 'm' drops from Bridge into water of the river. The bridge is 10m high from the water surface.

(a). Find the speed of the body 5m above the water surface.

(b). Find the speed of the body before it strikes the water.

((a) 9.9 ms^{-1} (b) 14 ms^{-1})

Given data:

Height of bridge = $h = 10 \text{ m}$

Height of point A = $h_1 = 5 \text{ m}$

Required:

(a) Speed 5m above surface = $V_A = ?$

(b) Speed just above water surface = $V_B = ?$ **Solution:**

When the body drops from the bridge then by work energy theorem

$$(P.E.)_{\text{loss}} = (K.E.)_{\text{gain}}$$

$$mgh_1 = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh_1} \quad \text{--- (i)}$$

(a) To find velocity " v_A " at point "A", $h_1 = 5\text{m}$ equation (i) becomes

$$v_A = \sqrt{2gh_1}$$

$$v_A = \sqrt{2(9.8)(5)}$$

$$v_A = 9.9 \text{ m/sec}$$

Result:(b) Now to find velocity " V_B " at point just above water surface then eq (i) becomes

$$v_B = \sqrt{2gh}$$

$$v_B = \sqrt{2(9.8)(10)}$$

$$v_B = 14 \text{ m/sec}$$

Result:**Problem. 10:** The engine of a JF-Thunder fighter (made by Pakistan and China) develops a thrust of 3000N. What horsepower does it has at a velocity of 600 m s^{-1} ? (2413 hp)**Given data:**

$$\text{Thrust of engine} = F_{th} = 3000 \text{ N}$$

$$\text{Velocity of engine} = V = 600 \text{ m/sec}$$

$$\text{Power} = P = ?$$

Required:**Solution:**

As we know that

$$\vec{P} = \vec{F}_{th} \cdot \vec{V}$$

$$P = F_{th} V \cos \theta$$

$$P = F_{th} V \cos 0$$

$$P = F_{th} V (1)$$

$$P = F_{th} V$$

By putting values

$$P = (3000)(600)$$

$$P = 1800000 \text{ watt}$$

Convert watt into hp

$$746 \text{ watt} = 1 \text{ hp}$$

Then

$$1 \text{ watt} = \frac{1}{746} \text{ hp}$$

$$1 \times 1800000 \text{ watt} = \frac{1800000}{746} \text{ hp}$$

Result:

$$P = 1800000 \text{ watt} = 2413 \text{ hp}$$

Problem. 11: The mass of the moon is $1/80$ of the mass of the earth and Corresponding radius is $1/4$ of the earth. Calculate the escape velocity on the surface of moon. (2.5 kms^{-1})**Given data:**

$$\text{Mass of moon} = M_m = \frac{1}{80} M_E$$

$$\text{Radius of moon} = R_m = \frac{1}{4} R_E$$

$$\text{Mass of earth} = M_E = 5.98 \times 10^{24} \text{ Kg}$$

$$\text{Radius of earth} = R_E = 6.4 \times 10^6 \text{ m}$$

$$\text{Gravitational constant} = G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

Required:

Escape velocity on moon surface = $V_{\text{esc(m)}} = ?$

Solution:

The escape velocity of moon is given by

$$V = \sqrt{\frac{2GM_m}{R_m}}$$

Putting values

$$V_{\text{esc(m)}} = \sqrt{\frac{2G \frac{1}{80} M_E}{\frac{1}{4} R_E}}$$

$$V_{\text{esc(m)}} = \sqrt{\frac{2(4)GM_E}{(80)R_E}}$$

$$V_{\text{esc(m)}} = \sqrt{\frac{GM_E}{10R_E}}$$

$$V_{\text{esc(m)}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{10(6.4 \times 10^6)}}$$

Result:

$$V_{\text{esc(m)}} = 2500 \text{ m/sec} = 2.5 \text{ km/sec}$$

Q. 1: What are centripetal acceleration and centripetal force? Derive their equations.

Ans. Centripetal Acceleration $\vec{a}$:

The acceleration due to change in the direction of velocity which is always directed towards the center of the circle is called centripetal (center-seeking) acceleration.

Nature:

It is a vector quantity and always directed towards the center of the circle in which the body moves.

Dimensions:

The dimensions of centripetal acceleration are $[LT^{-2}]$

$$\text{As } \vec{a}_c = \frac{V^2}{r} \hat{r}$$

$$[\vec{a}] = \left[\frac{V^2}{r} \right]$$

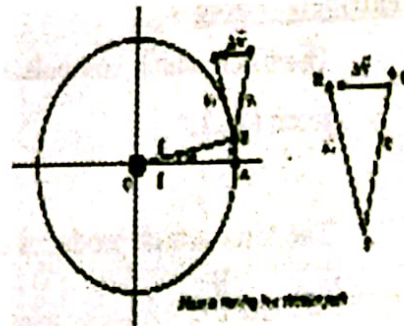
$$[\vec{a}] = \frac{[LT^{-1}]^2}{[L]}$$

$$[\vec{a}] = \left[\frac{L^2 T^{-2}}{L} \right]$$

$$[\vec{a}] = [LT^{-2}]$$

Derivation of Equation for Centripetal Acceleration $\vec{a}_c$:

Consider a mass "m" moving with "v" in a circle of radius "r" as shown in figure. The velocity of mass "m" at time t_1 is $\vec{V}_1 = \overrightarrow{PQ}$ and at time t_2 is $\vec{V}_2 = \overrightarrow{PR}$. When Δt is very small it implies that point B is close to point A and arc AB is nearly equal to $\overline{AB}$. These are isosceles triangles, so the ratio of the corresponding sides are equal hence:



$$\frac{AB}{AC} = \frac{QR}{PQ} \quad \text{Since}$$

$$\text{OR } \frac{S}{r} = \frac{\Delta V}{V} \text{ --- (i) as}$$

$$\text{Also } S = V\Delta t \quad (\text{As } S = Vt)$$

$$\Delta t \ll \ll$$

$$\vec{V}_1 = \vec{V}_2 = \vec{V}$$

$$AB = S \text{ and } AC = r$$

$$\overline{QR} = \Delta V, \overline{PQ} = V$$

$$(i) \Rightarrow \frac{V\Delta t}{r} = \frac{\Delta V}{V}$$

$$\frac{\Delta V}{\Delta t} = \frac{V^2}{r}$$

$$\text{OR } \bar{a}_c = \frac{v^2}{r} \quad \therefore \frac{\Delta \bar{v}}{\Delta t} = \bar{a}$$

$$\text{OR } \bar{a}_c = \frac{-v^2}{r} \hat{r}$$

The negative sign shows that centripetal acceleration is always directed towards the center of the circular path.

Vector Form of $\bar{a}_c$:

$$\bar{a}_c = \frac{-v^2}{r} \hat{r} \quad \text{--- (ii)} \quad \text{But } \bar{r} = r \hat{r}$$

$$\Rightarrow \bar{a}_c = \frac{-v^2}{r} \frac{\bar{r}}{r} \quad \text{and } \hat{r} = \frac{\bar{r}}{r}$$

$$\Rightarrow \boxed{\bar{a}_c = \frac{-v^2}{r^2} \bar{r}}$$

Angular Form of $\bar{a}_c$:

$$\text{As } a_c = \frac{-v^2}{r}$$

$$\text{But } v = r\omega$$

Put in above equation,

$$\bar{a}_c = \frac{-(r\omega)^2}{r}$$

$$\bar{a}_c = \frac{-r^2\omega^2}{r}$$

$$\bar{a}_c = -\omega^2 r$$

Centripetal Force $\bar{F}_c$:

The force which compels a body to move in circular path is called centripetal force ($\bar{F}_c$).

OR

The force which produces centripetal acceleration in a body.

Nature:

It is a vector quantity and it is directed towards the center of the circular path.

Derivation of Equation $\bar{F}_c$:

According to Newton's second law of motion:

$$F = ma$$

$$\text{OR } F_c = ma_c$$

$$\text{But } \bar{a}_c = \frac{-v^2}{r} \hat{r}$$

$$\text{Hence } \bar{F}_c = \frac{-mv^2}{r} \hat{r}$$

- i) During turn or vehicle friction behaves as centripetal force.
- ii) In a whirling stone tied to string tension behaves as centripetal force.

Vector Form of $\vec{F}_c$:

As $\vec{F}_c = \frac{-mv^2}{r} \hat{r}$ But $r = \frac{\vec{r}}{r}$

Put in above equation

$$\Rightarrow \vec{F}_c = \frac{-mv^2}{r} \frac{\vec{r}}{r}$$

$$\vec{F}_c = \frac{mv^2}{r^2} \vec{r}$$

This is the vector form of centripetal force

Angular Form:

Also $\vec{F}_c = \frac{-mv^2}{r} \hat{r}$ but $v = r\omega$

$$\Rightarrow \vec{F}_c = \frac{m(r\omega)^2}{r} \frac{\vec{r}}{r}$$

$$\text{Or } \vec{F}_c = \frac{mr^2\omega^2}{r^2} \vec{r}$$

$$\text{Or } \vec{F}_c = m\omega^2 \vec{r}$$

This is the angular form of centripetal force.

Q. 2: Show that angular momentum in magnitude is given by: $|\vec{L}| = |\vec{r} \times \vec{p}| = mr^2\omega = mvr$

Ans. Angular Momentum:

"The moment of linear momentum is called angular momentum".

OR

The cross (vector) product of position vector " $\vec{r}$ " and linear momentum $\vec{p}$ is called angular momentum. i.e. $\vec{L} = \vec{r} \times \vec{p}$

Nature:

Angular momentum is a vector and according to definition of vector product its direction is always perpendicular to $\vec{r}$ and $\vec{p}$.

Dimensions of $\vec{L}$:

Dimensions of angular momentum are:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$[\vec{L}] = [\vec{r} \times \vec{p}]$$

$$[\vec{L}] = [L \times M L T^{-1}]$$

$$[\vec{L}] = [M L^2 T^{-1}]$$

Remember that the dimensions and units of angular momentum $\vec{L}$ and planck constant "h" are same.

Unit:

The unit of angular momentum is $kg.m^2/s$ which is equivalent to J.S (Joule Second).

To Show That $\vec{L} = [\vec{r} \times \vec{p}] = m r^2 \omega = m v r$:

Consider a mass "m" is moving with velocity $\vec{V}$ in a circle of radius r as shown. The linear velocity $\vec{V}$ and $\vec{p}$ are tangential to the circular path. The radius r and $\vec{p}$ are in xy-plane but the direction of angular momentum is perpendicular, along the axis of rotation either in or out of the plane of page depending upon the sense of rotation. It can be seen from the figure that angular momentum depends upon the product of $\vec{p}$ and $r_{\perp}$ (perpendicular components of r) i.e.

$$L = r_{\perp} p \text{-----(i)}$$

But

$$r_{\perp} = r \sin \theta$$

Equation (i) becomes as

$$L = (r \sin \theta) p$$

$$L = (\text{The magnitude of the component of } r \perp \text{ to } p) p$$

Which is the definition of vector product i.e. $\vec{L} = \vec{r} \times \vec{p}$

To Show That

$$\vec{L} = [\vec{r} \times \vec{p}] = m v r$$

As

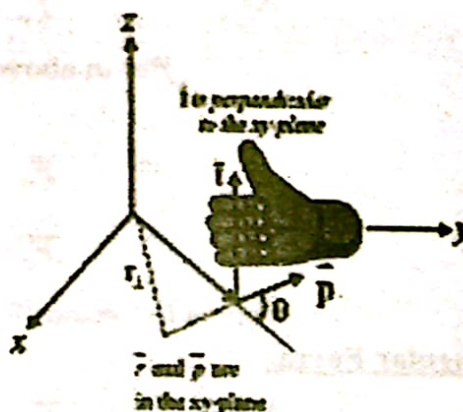
$$\vec{L} = \vec{r} \times \vec{p}$$

Its magnitude when $\theta = 90^\circ$ is given as:

$$|L| = |\vec{r} \times \vec{p}| = r p$$

But $p = mv$ (definition of $\vec{p}$)

Put in above equation



In three-dimensional space, the position vector $\vec{r}$ becomes a particle in the xy-plane with linear momentum $\vec{p}$. The angular momentum with respect to the origin is $\vec{L} = \vec{r} \times \vec{p}$ which is in the z-direction. The direction of $\vec{L}$ is given by the right-hand rule.

$$|\vec{L}| = |\vec{r} \times \vec{p}| = r(mv)$$

OR $|\vec{L}| = |\vec{r} \times \vec{p}| = mvr$

Hence proved.

To Show That $\vec{L} = \vec{r} \times \vec{p} = mr^2\omega$

As $|\vec{L}| = |\vec{r} \times \vec{p}| = mvr$

But $v = r\omega$

Put in above equation

$$|\vec{L}| = |\vec{r} \times \vec{p}| = m(r\omega)r$$

OR $|\vec{L}| = |\vec{r} \times \vec{p}| = mr^2\omega$

Hence the required proof.

Q.3: Show that role playing by mass in linear motion is playing by moment of inertia in rotatory motion.

Ans. **Moment of Inertia:**

The product of mass and squared radius (position vector) is called moment of inertia.

Mathematically:

$I = mr^2$ It is the rotational analogue of mass m and its unit is kgm^2 .

Proof:

Consider a rod is pivoted at point "O" and force $\vec{F}$ acts on the rod. The torque acting on the rod is given as:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin \theta, \quad \theta = 90^\circ$$

$$\tau = rF \sin 90^\circ$$

$$\tau = rF(1)$$

$$\tau = rF \text{-----(i)}$$

But $F = ma$ and $a = r\alpha$

So $F = m(r\alpha)$

Put this value in (i)

$$\tau = r(mr\alpha)$$

$$\tau = mr^2\alpha$$

$$\tau = I\alpha \text{-----(ii) Where } I = mr^2$$

Compare (ii) with Newton's second law i.e. $F = ma$ it shows that $I(mr^2)$ is the rotational analogue of m as α and τ are the rotational analogue of a and $\vec{F}$ respectively.

Q. 4: What do you mean by "INTELSAT"? At what frequencies it operates? For how many TV stations this system is used?

Ans. **INTELSAT:**

It is an acronym for an organization of international telecommunication satellites. It is the largest telecommunication satellite system of the globe (دنیا).

Historical Backgrounds of "INTELSAT":

The idea of the creation of the international telecommunication satellites organization was presented in 1961.

This was originally formed as international telecommunication satellite organization (ITSO).

It started its telecommunication operations in 1964.

Objective of Establishment:

Its purpose of establishment is to provide reliable, high quality international public telephony on a non-discriminatory (بلا تفریق) basis to all areas of the world through development of a global commercial telecommunication satellite system.

Frequencies of Operation of "INTELSAT":

It operates at frequencies of 4, 6, 11 and 14 GHz where $1\text{GHz} = 10^9\text{Hz}$

T.V Stations:

As of 2018, "INTELSAT" provides service to over 600 earth stations in more than 149 countries and has a capacity of 30000 two way telephone circuits with three TV channels.

Q. 5: Show that in angular form, centripetal acceleration is $a_c = -\omega^2 r$

Ans See long question No. 1.

Q. 6: Show that centripetal force is also shown by $\vec{F}_c = \frac{-mv^2}{r} \hat{r} = -m\omega^2 \vec{r} = \frac{-mv^2}{r^2} \vec{r}$

Ans See long question No. 1.

Q. 7: Show that a satellite near the earth will have greater velocity.

Ans. **To Show That the Satellite Near the Earth will have Greater Velocity:**

Consider a satellite of mass " m_s " is orbiting around earth in an orbit of radius " r " with speed " v " as shown in figure. The necessary centripetal force $\vec{F}_c$ is provided by gravitational force of earth.

$$\text{i.e. } \vec{F}_c = \vec{F}_g \quad \text{OR} \quad \frac{m_s v^2}{r} = \frac{G m_e m_s}{r^2}$$

Where $m_e = \text{mass of earth}$

||| I ... will get:



But $\sqrt{Gm_e} = \text{Constant}(s)$

Hence $v = \frac{\text{Constant}}{\sqrt{r}}$

OR $v \propto \frac{1}{\sqrt{r}}$

This equation shows that velocity of satellite and radius of orbit in which satellite orbits are inversely proportional. Hence if a satellite is near to earth i.e. smaller the "r" greater will be the velocity of satellite.

Q. 8: What do you mean by weight of a body? Use examples to distinguish between real weight and the apparent weight of a body.

Weight of a Body:

The pull of gravity on a body is called its weight.

OR

The force applied by earth on a body is called its weight.

Nature of Weight:

Weight is a vector quantity and its direction is always toward the center of earth.

Dimensions of Weight:

Since weight is a force so its dimensions are that of force that are $[MLT^{-2}]$.

Weight on or Near Earth's Surface:

To calculate the weight near earth's surface the relation $W = mg$ is used

Weight at Altitude "h" Above Earth's Surface:

The weight of a body at certain height "h" above earth's surface is calculated

by" $F = \frac{GmM_e}{(R_e + h)^2}$

where $M_e = \text{Mass of earth} = 6.0 \times 10^{24} \text{ kg}$

$m = \text{Mass of body}$

$R_e = \text{Radius of earth} = 6.4 \times 10^6 \text{ m}$

$h = \text{Height above earth's surface}$

$G = 6.67 \times 10^{-11} \text{ N.m}^2 / \text{kg}^2$

Real Weight:

The pull of earth on a body is called real weight.

Apparent Weight:

The force which prevents a body from falling is called apparent weight of a body.

Distinction (فرق) of Real and Apparent Weight with Examples:

First Example:

Consider a block is placed on the ground as shown in figure. The force of earth ground is real weight of the block (body) but the reaction of the ground surface on the block is its apparent weight.

Second Example:

Consider a ball of mass "m" is suspended from a string as shown. The pull of earth is the real weight but the tension (T) in the string is the apparent weight of the ball which prevents it from falling. Since the ball is in equilibrium the real and apparent weights of ball are equal. i.e.

$$W_{\text{real}} = W_{\text{apparent}}$$

Third Example:

Consider a person observing a fish hanging from a spring balance in a lift the forces acting on it are given by $F = W - T$ then there are the following cases.

i) **When the Lift is at Rest:**

When the lift is at rest the person and lift are in equilibrium i.e.

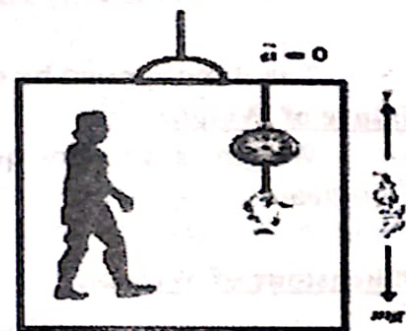
$$F_{\text{net}} = 0$$

$$\text{Hence } F = W - T$$

$$0 = W - T$$

$$\text{OR } W = T$$

Hence the real and apparent weights of the fish are same.



(a) When the lift is at rest, the spring balance reads the real weight of the fish.

ii) **When the Lift is Moving Upwards:**

In this case tension (T) in the string is effective or net force hence

$$F_{\text{net}} = T - W$$

$$\text{OR } T = F_{\text{net}} + W \text{ OR } T > W$$

This shows that apparent weight i.e. tension (T) is greater than real weight.

iii) **When the Lift is Moving Downwards:**

In this case weight is effective or net force hence

$$F_{\text{net}} = W - T$$

$$\text{OR } T = W - F_{\text{net}}$$

$$\text{OR } T < W$$

This shows that apparent weight (T) is smaller than real weight (W).

iv) **When the Lift Falls Freely:**

In this case acceleration is gravitational acceleration

$$\text{i.e. } F_{\text{net}} = ma = mg$$

$$\text{As } F_{\text{net}} = W - T$$

$$\text{OR } T = W - F_{\text{net}}$$

$$T = mg - mg$$

OR $T = 0$

This shows that if a body falls freely its apparent weight is zero.

Conclusion:

The apparent weight of a body can be equal, smaller, and greater than the real weight of the body and even it can become zero.

Q. 9: Explain, how gravity is provided to the occupants of the spaceship?

Ans. Providing Artificial Gravity to the Occupants of Spaceship:

Introduction:

Gravity is provided to the occupants of spaceship by turning or rotating the spaceship around its own axis. The rotation of spaceship gives rise to centrifugal (reaction) force which acts on the occupants of satellite similar to the weight of a body on earth.

Explanation:

i) Construction:

Spaceships are usually built in the form of large wheels with hollow rims. The outer rim behaves as a floor.

ii) Artificial Gravity:

The gravity which is provided by making the spaceship to rotate around its own axis is called artificial (un-natural) gravity. The value of artificial gravity depends upon the angular speed of spaceship.

iii) What's the Need of a Artificial Gravity:

The spaceship revolves around the earth in a fixed orbit and does not fall to earth. Consequently the apparent weight of spaceship and occupants becomes zero. The occupants feel weightless in the spaceship. Due to weightlessness the action and reaction forces disappear and it becomes difficult for the astronauts to move or walk properly. Hence to carry out their work properly, artificial gravity is produced.

iv) Mathematical Treatment:

For the orbiting of spaceship

$$V = rw \quad \text{And} \quad a_c = g$$

But $a_c = \frac{v^2}{r}$, Put $v = rw$

$$a_c = \frac{(rw)^2}{r}$$

$$a_c = \frac{r^2 w^2}{r}$$

$$a_c = rw^2 \quad \text{but} \quad a_c = g$$

Hence $g = rw^2 \text{ ----- (i)}$

Also $S = vt$ or $t = \frac{S}{v}$

But $S = 2\pi r$ and $t = T$

So $T = \frac{2\pi r}{v} = \frac{2\pi f}{f w} = \frac{2\pi}{w}$

OR $w = \frac{2\pi}{T}$ or $\left(\text{as } \frac{1}{T} = f \right)$

OR $w = 2\pi f$

Put this value of w in equation (i)

$$g = r(2\pi f)^2$$

$$g = r 4\pi^2 f^2$$

OR $f^2 = \frac{g}{4\pi^2 r}$

$$f = \sqrt{\frac{g}{4\pi^2 r}}$$

OR $f = \frac{1}{2\pi} \sqrt{\frac{g}{r}}$

Hence the artificial gravity depends upon

- Radius of spaceship
- Frequency of revolution of spaceship

Q. 10: Give different three examples to illustrate the phenomenon of conservation of angular momentum.

Ans. Conservation of Angular Momentum:

If the net torque acting on a system is zero the total angular momentum of the system remains constant or conserved.

i.e. $L = \text{constant}, \quad \tau = 0$

OR $\Delta L = 0$

$L_f - L_i = 0$

OR $L_f = L_i$

Other Way:

Since $\tau = \frac{\Delta L}{\Delta t}$

As $\tau = 0 \Rightarrow \frac{\Delta L}{\Delta t} = 0$

But $\Delta t \neq 0 \Rightarrow \Delta L = 0$

OR $L_f - L_i = 0$

OR $L_f = L_i$

OR $mr_f^2 w_f = mr_i^2 w_i$

Illustration by Example:

effective radius and moment of inertia (I_f) of his body.

As $L_i = L_f$

OR $mr_i^2 \omega_i = mr_f^2 \omega_f$

If $r_i > r_f$ then $\omega_i < \omega_f$

And if $r_i < r_f$ then $\omega_i > \omega_f$

- ii) When a gymnast starts dismount, he completely extend his body to increase effective radius r_i and moment of inertia I_i which decreases his angular speed ω_i . But the gymnast then tucks in his knees to decrease the effective radius r_f and moment of inertia I_f which increases his angular speed ω_f due to conservation of angular momentum.

As $mr_i^2 \omega_i = mr_f^2 \omega_f$

If $r_i > r_f$ or $I_i > I_f$

Then $\omega_i < \omega_f$

But if $r_i < r_f$ or $I_i < I_f$

Then $\omega_f > \omega_i$

Increasing his angular speed ω_f enables him to complete rotation.

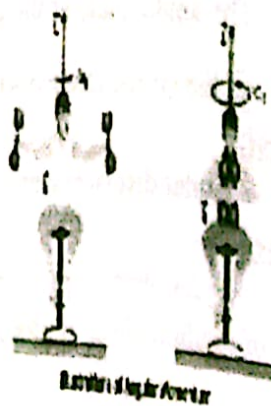
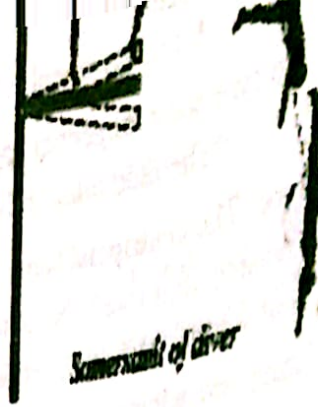
- iii) A man standing on the turn table starts moving with arms stretched horizontally to increase the effective radius r_i and moment of inertia I_i and hence move with smaller speed ω_i . When he draws his hands in towards chest the effective radius r_f and moment of inertia I_f decreases, which increases his angular velocity ω_f

As $mr_i^2 \omega_i = mr_f^2 \omega_f$

If $r_i > r_f$ or $I_i > I_f$

Then $\omega_i < \omega_f$

But if $r_i < r_f$ or $I_i < I_f$



Ans. The mud guards are used on the wheels of cycles, motor cars and other driving vehicles to prevent the falling of mud and other waste lying on the roads, on other parts of vehicles, near wheels.

Explanation:

When the wheel rotates each point on the tyre and muds or wastes clinging to the tyre has a tangential speed $\vec{V}$ as well as angular momentum.

The mud cling to tyre due to the presence of adhesive forces between mud and tyre. The centripetal force $\vec{F}_c$ is provided to mud by the adhesive forces between mud (wastes) and tyre. The centripetal force acts radially outward due to the inertia of mud and wastes. When the speed of the vehicle is low, the centrifugal force is smaller and then the adhesive forces and the mud and wastes remain clinging to the tyre. But when the speed of vehicle increases to a certain limit, the centrifugal force overcome adhesive force and mud and other wastes fly tangentially and strike the mud guards.

Conclusion:

Hence the mud guards prevent the mud and other wastes not to reach the other surrounding or nearby parts of the vehicle.

Non-Exercise Long Questions

Q. 1: Define angular motion and angular displacement. Show that $S = r\theta$

Ans. **Angular Motion:**

The motion of a body about ($\curvearrowright$) a fixed point or fixed axis, is called angular motion.

Examples:

- i) Motion of moon around earth
- ii) Motion of electron around nucleus
- iii) Motion of wheel around axle
- iv) Motion of earth around sun

The line around which a body rotates is called axis of rotation.

Angular Displacement " θ ":

The angle through which a body moves on a circular path.

OR

The angle made at the center of a circle by the position vector ($\vec{r}$) of a point on the circumference with initial position, is called angular displacement.

Symbol:

Angular displacement is denoted by the symbol θ .

Nature:

Angular displacement is a vector because it has a sense of rotation either clockwise or anti-clockwise. It is



negative in the direction of decreasing angle.

Unit: The unit of angular displacement is called radian.

Numerical Value of Radian: The numerical value of radian is 57.3°

Definition of 1 Radian:

The angle subtended by an arc at the center of a circle, equal in length to the radius of that circle, is called one radian. i.e.

$$\hat{S} = \bar{r}(\text{radius}), \quad \theta = 1 \text{ radian}$$

Dimensions of Angular Displacement " θ ":

Angular displacement is a dimensionless quantity, it has no dimensions.

To show $S = r\theta$:

Consider the position vector of a body has moved from point A to B along the circumference of a circle making an angle $\angle AOB$ at the center. Let it moves to point "C" making angle $\angle AOC$ such that the length of arc AC is equal to radius "r" of this circle it is evident from figure that:

$$S \propto \angle AOB$$

$$\text{OR } S = k_1 \angle AOB \text{ ----- (i)}$$

$$\text{Also } r \propto \angle AOC$$

$$\text{OR } r = k_2 \angle AOC \text{ -----(ii)}$$

Since the arcs and angles are proportional their ratios are equal

$$\text{i.e. } k_1 = k_2 = k$$

Divide equation (i) by (ii)

$$\frac{S}{r} = \frac{k_1 \angle AOB}{k_2 \angle AOC}$$

$$\text{OR } \frac{S}{r} = \frac{k \angle AOB}{k \angle AOC} \quad \text{as } k_1 = k_2 = k$$

$$\frac{S}{r} = \frac{\angle AOB}{\angle AOC} \text{ -----(ii)}$$

But $\angle AOC = 1 \text{ rad}$, because arc AC is equal to radius of circle,

$$\text{Also } \angle AOB = \theta$$

$$\text{So } \frac{S}{r} = \frac{\theta}{1}$$

$$\Rightarrow S(1) = r(\theta)$$

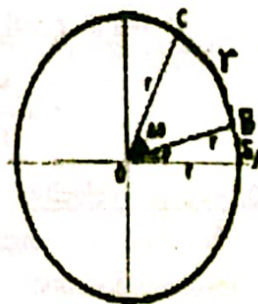
$$\Rightarrow S = r\theta$$

$$\text{If } S = 2\pi r, \quad \theta = 360^\circ$$

$$\text{Hence } 2\pi r = r 360^\circ$$

$$2\pi \text{ rad} = 360^\circ$$

$$1 \text{ rad} = \frac{360^\circ}{2\pi}$$



Ans. **Angular Velocity " ω ":**

The angular displacement θ covered by a body in one second is called angular velocity.

OR

The rate of change of angular displacement is called angular velocity

Symbol:

Its symbol ω

Mathematically:

$$\text{Angular velocity} = \frac{\text{Angular Displacement}}{\text{time}}$$

$$\bar{\omega} = \frac{\bar{\theta}}{t}$$

$$\text{OR } \bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\text{OR } \bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

But if $t_i = 0$, $\theta_i = 0$

$$\bar{\omega} = \frac{\theta_f}{t_f} \quad \text{or} \quad \bar{\omega} = \frac{\theta}{t}$$

Unit of Angular Velocity:

The unit of angular velocity is radians/second, degree/second or revolutions/second.

Nature of Angular Velocity:

Angular velocity is a vector quantity and its direction is always along the axis of rotation. The direction of angular velocity is determined by right hand rule.

Dimensions of Angular Velocity:

The dimensions of angular velocity are $\left[\frac{1}{T}\right]$ or $[T^{-1}]$ angular velocity is also called as angular frequency. Its relation with linear frequency "f" is as: $\omega = 2\pi f$

To Show That $\vec{r} = \bar{\omega} \times \vec{r}$:

Consider a particle of mass "m" moves along a circular path as shown in

figure the angular displacement θ is given by $\theta = \frac{S}{r}$

$$\text{OR } S = r\theta$$

If S changes by an amount ΔS , θ changes by $\Delta\theta$.

$$\text{Hence } \Delta S = r\Delta\theta \quad (r = \text{constant})$$

Divide both sides by Δt

$$\frac{\Delta S}{\Delta t} = r \frac{\Delta\theta}{\Delta t} \text{-----}(i)$$

As $\frac{\Delta S}{\Delta t} = V, \quad \frac{\Delta \theta}{\Delta t} = \omega$

(i) $\Rightarrow v = r\omega$

In vector form

$$\vec{v} = \vec{\omega} \times \vec{r}$$

The direction of $\vec{\omega}$ is perpendicular to linear (tangential) velocity $\vec{v}$ as well as radius of the circular path.

Q.3: Define angular acceleration $\vec{\alpha}$ show that $\vec{a} = \vec{\alpha} \times \vec{r}$.

Angular Acceleration " $\vec{\alpha}$ ":

"The rate of change of angular velocity is called angular acceleration".

Symbol:

Its symbol is $\vec{\alpha}$ (alpha).

Mathematically:

$$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$$

Nature:

It is a vector and its direction is always along the axis of rotation. If the angular velocity increase the direction of angular acceleration $\vec{\alpha}$ and angular velocity are same.

If the angular velocity decrease, the direction of angular acceleration $\vec{\alpha}$ and angular velocity $\vec{\omega}$ are opposite to each other.

Unit:

The unit of angular acceleration is *radian / S², degree/S², revolution/S²*.

Dimensions of " $\vec{\alpha}$ ":

The dimensions of $\vec{\alpha}$ are $[T^{-2}]$.

To Show That $\vec{a} = \vec{\alpha} \times \vec{r}$:

According to definition of linear acceleration $\vec{a}$

$$\vec{a} = \frac{\Delta \vec{V}}{\Delta t}$$

OR

$$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t} \quad \text{but} \quad \vec{V} = \vec{\omega} \times \vec{r}$$

$$\Rightarrow \frac{(\omega_f - \omega_i) \times r}{\Delta t}$$

$$\Rightarrow \frac{\omega_f \times r - \omega_i \times r}{\Delta t}$$

$$\Rightarrow \vec{a} = \frac{\Delta \vec{\omega}}{\Delta t} \times \vec{r}$$

But

$$\frac{\Delta \vec{\omega}}{\Delta t} = \vec{\alpha}$$

$$\Rightarrow \boxed{a = \alpha \times r}$$

Which is the required relation between linear acceleration " $\vec{a}$ " and angular acceleration " $\vec{\alpha}$ ".

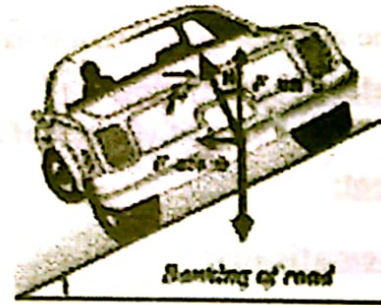
4: Explain the banking of roads.

5. **Banking of Roads:**

The phenomenon in which outer edges of curved roads are raised with respect to the inner edges to provide the necessary centripetal force to the vehicles to take a safe turn is called banking of roads.

How Does Banking Help?

The frictional force between tyres of the vehicle and road may sometimes not be enough to provide necessary centripetal force for the safe turn of vehicle. Specially on rainy day, oily road or when the condition of the tyre is not good enough to provide required amount of friction.



To overcome this problem banking of roads help to make the turn of vehicle safe. The reaction of the outer bank of roads provides the necessary centripetal force needed for safe turn.

Mathematically:

If the vehicle is considered as a particle, then the figure shows that horizontal component $F \sin \theta$ acts inward to the center of the turn (circle) which provides the necessary centripetal force.

As $\vec{F}_c = F \sin \theta$

$$\frac{mv^2}{r} = F \sin \theta \text{ ----- (i)}$$

Also $mg = F \cos \theta \text{ ----- (ii)}$

Because $F \cos \theta$ balances the weight and acts upward as shown.

Divide (i) by (ii)

$$\frac{\frac{mv^2}{r}}{mg} = \frac{F \sin \theta}{F \cos \theta}$$

$$\frac{mv^2}{r} + \frac{mg}{1} = \tan \theta$$

OR $\frac{mv^2}{r} \times \frac{1}{mg} = \tan \theta$

$$\frac{v^2}{rg} = \tan \theta$$

OR $\tan \theta = \frac{v^2}{rg} \quad \text{or} \quad \theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$

For a given value of $\vec{r}$ the banking angle θ works for a particular value of velocity $\vec{V}$.

5: Define and explain moment of inertia of a single body and rigid body.

OR

Discuss the relation between torque and moment of inertia of a body.

Ans. **Moment of Inertia:**

The property of a body due to which it opposes a change in its state of rest and uniform angular motion is called moment of inertia.

It is also called as rotational inertia.

OR

The product of mass and squared position vector (radius r) is called moment of inertia.

Mathematically:

$$I = (\text{mass})(\text{radius})^2$$

$$I = mr^2$$

Nature:

It is a scalar quantity.

Unit:

Its unit is kg.m^2 .

Dimensions:

Its dimensions are $[ML^2]$.

Factors Upon Which Moment of Inertia:

- i) Mass of body
- ii) Radius of the body
- iii) Distribution of mass

Derivation of Equation for Moment of Inertia:

Consider a rod of mass "m" pivoted at point "O" acted upon by a force "F".

The torque produced due to $\vec{F}$ is given as:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

OR

$$|\vec{\tau}| = rF \sin \theta$$

But

$$\theta = 90^\circ, \quad \sin 90^\circ = 1$$

Hence

$$\tau = rF \sin 90^\circ$$

$$\tau = rF(1)$$

$$\tau = rF$$

But $F = ma$ Newton's second law

$$\tau = r(ma)$$

But

$$a = r\alpha$$

$$\tau = rm(r\alpha)$$

$$\Rightarrow \tau = (mr^2)\alpha$$

$$\tau = I\alpha$$

Where $I = mr^2$ is called moment of inertia of body.

Moment of Inertia of Rigid (☞) Body:

A rigid body is that in which the distance among particles does not change if external force is applied.



Consider a rigid body having mass $m_1, m_2, \dots, m_n$ radii $r_1, r_2, r_3, \dots, r_n$ as shown in figure. The moment of inertia is $I_1, I_2, I_3, \dots, I_n$ are given by:

$$I_1 = m_1 r_1^2, \quad I_2 = m_2 r_2^2$$

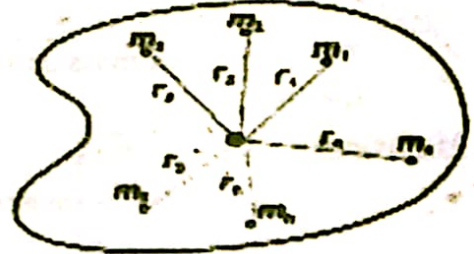
$$I_3 = m_3 r_3^2, \dots, I_n = m_n r_n^2$$

The inertia of the whole rigid body is:

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$

$$\text{OR} \quad I = \sum_{i=1}^n m_i r_i^2$$



Which is the moment of inertia of a rigid body.

Q. 6: Explain angular momentum of rigid body. Show that the rate of change of angular momentum is equal to torque acting on that body.

Ans. Consider a rigid (مخت) body of many number of particles, of masses, $m_1, m_2, \dots, m_n$ having radii $r_1, r_2, \dots, r_n$ as shown in figure.

The angular momentum of mass, m_1 and radius r_1 is $\bar{L}_1$ given by:

$$L_1 = m_1 r_1^2 \omega$$

$$L_2 = m_2 r_2^2 \omega \quad (\omega \text{ is same for all masses})$$

$$L_n = m_n r_n^2 \omega$$

Total angular momentum of rigid body is equal to the sum of angular momenta of all masses, i.e.

$$L = L_1 + L_2 + L_3 + \dots + L_n$$

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + \dots + m_n r_n^2 \omega$$

$$L = (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2) \omega$$

$$\text{OR} \quad L = \sum_{i=1}^n (m_i r_i^2) \omega$$



This is the angular momentum of rigid body. Where $\sum_{i=1}^n m_i r_i^2$ is the total moment of inertia of rigid body. Thus total angular momentum $\bar{L}$ can also be expressed as $\bar{L} = I \bar{\omega}$

This equation is the rotational analogous equation for $\bar{p} = m \bar{v}$

To Show That $\frac{\Delta \bar{L}}{\Delta t} = \bar{\tau}$:

From the definition of angular momentum:

$$\bar{L} = \bar{r} \times \bar{p}$$

OR $L = rps \sin \theta$ if $\theta = 90^\circ$

$$L = rp$$

OR $L = rmv$

A change in velocity will cause a change in angular momentum because m and r are constants, i.e.

$$\Delta L = rm \Delta v$$

Divide both sides by Δt

$$\frac{\Delta L}{\Delta t} = \frac{rm \Delta v}{\Delta t}$$

OR $\frac{\Delta L}{\Delta t} = rm \frac{\Delta v}{\Delta t}, \quad \left(\text{but } \frac{\Delta v}{\Delta t} = \bar{a} \right)$

So $\frac{\Delta L}{\Delta t} = rma$

OR $\frac{\Delta L}{\Delta t} = r(ma) \quad \text{Where } ma = F$

So $\frac{\Delta L}{\Delta t} = rF \quad \text{Where } rF = \tau$

So $\frac{\Delta L}{\Delta t} = \bar{\tau}$

This shows that the rate of change of angular momentum of a body is equal to the net torque acting on the body. This equation $\frac{\Delta L}{\Delta t} = \bar{\tau}$ is the rotational analogous equation for $\frac{\Delta P}{\Delta t} = \bar{F}$.

Q. 7: Define rotational or angular K.E. Derive relations for angular K.E of rigid body.

Ans. Rotational/Angular K.E:

The energy of a body due to its rotational or angular motion is called rotational or angular K.E.

Mathematically:

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

Where I = moment of inertia

ω = Angular velocity

As linear K.E of a body is given as:

$$K.E = \frac{1}{2} mv^2$$

Put $v = r\omega$ in above equation

$$K.E = \frac{1}{2} m (r\omega)^2$$

$$K.E = \frac{1}{2} mr^2 \omega^2$$

OR

$$K.E = \frac{1}{2} (mr^2) \omega^2$$

$$K.E = \frac{1}{2} I \omega^2 \quad (\text{Where } I \text{ is angular inertia})$$

K.E of a single body or particle.

rigid of masses, $m_1, m_2, \dots, m_n$ having radii $r_1, r_2, \dots, r_n$ as
The rotational K.E of $m_1, m_2, \dots, m_n$ are given by:

$$K.E_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$

$$K.E_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$

$$K.E_n = \frac{1}{2} m_n r_n^2 \omega^2$$

The total K.E of the rigid body is equal to the sum of
K.E, of all the masses.

$$K.E_T = K.E_1 + K.E_2 + \dots + K.E_n$$

$$K.E_T = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

Take $\frac{1}{2} \omega^2$ common

$$K.E_T = \frac{1}{2} \left(\sum_{i=1}^n m_i r_i^2 \right) \omega^2$$

Which is the total rotational or angular K.E of rigid body. Where $\sum_{i=1}^n m_i r_i^2$ is the total inertia of rigid body. The above equation can also be expressed as $K.E_T = \frac{1}{2} I \omega^2$. This equation is the rotational/angular analogous equation for equation $K.E = \frac{1}{2} m v^2$

Total K.E of Body:

The total K.E of a body is equal to the sum of its linear K.E and rotational K.E
i.e.

$$K.E_T = K.E_{rot} + K.E_{tran}$$

disc and hoop are freed from same level to roll down an inclined plane,
n first?

and or thin walled cylinder is called hoop, where a solid cylinder

it has translational (linear) K.E as well as rotational (angular)

$$K.E_T = K.E_{tran} + K.E_{rot}$$

conservation of energy the loss in P.E of disc is equal



Loss in P.E = Gain in K.E

$$mgh = K.E_{trans} + K.E_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Put moment of inertia as $I = \frac{1}{2}mr^2$

$$\omega = \frac{v}{r} \text{ In above equation}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{1}{2}mr^2 \frac{v^2}{r^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \text{ (m cancels out)}$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2$$

$$gh = \left(\frac{1}{2} + \frac{1}{4}\right)v^2$$

$$gh = \frac{3}{4}v^2$$

OR

$$v^2 = \frac{4gh}{3}$$

OR

$$v = \sqrt{\frac{4}{3}gh}$$

OR

$$v = \sqrt{\frac{4}{3}gh} \text{ ----- (i)}$$

This is the velocity of disc at bottom.

K.E of Hoop:

As a hoop rolls it has translational K.E as well as rotational K.E. i.e.

$$K.E = K.E_{trans} + K.E_{rot}$$

According to the law of conservation of energy, the loss in P.E of hoop is equal to gain in K.E of hoop.

Loss in P.E = Gain in K.E

$$mgh = K.E_{trans} + K.E_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Put $I = mr^2$ as rotational inertia for hoop and $\omega = \frac{v}{r}$ in above equation.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr^2 \frac{v^2}{r^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr^2$$



Rolling down of cylinder and hoop on an inclined plane

$$gh = \left(\frac{1}{2} + \frac{1}{2}\right)v^2 \quad (m \text{ cancels out})$$

$$gh = (1)v^2$$

$$v^2 = gh$$

$$v = \sqrt{gh} \quad \text{OR} \quad v = \sqrt{gh}$$

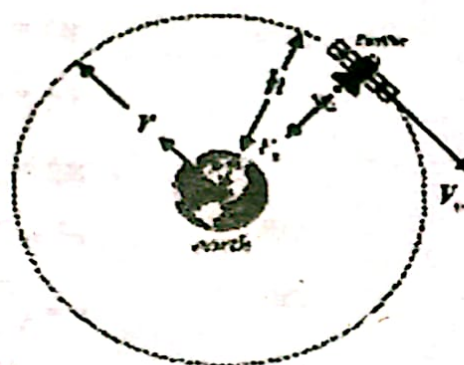
$$\text{OR} \quad v = \sqrt{gh} \text{-----(ii)}$$

This is the velocity of hoop at bottom of inclined plane. Compare equation (i) and (ii) it can be seen that $\sqrt{\frac{4}{3}}$ is greater than unity (1) hence the velocity of disc is higher than that of hoop. Hence the disc will reach the bottom first.

Q. 9: Explain weightlessness in satellite, define critical velocity.

Ans. Weightlessness in Satellite:

A satellite orbiting around earth moves in an orbit of nearly fixed radius "r" with velocity " $\vec{v}$ " as shown. If orbiting satellite is attached to a spring balance, the spring balance would show zero reading. Because satellite does not fall towards earth although attracted by earth, it does not stretch or extend the spring of the spring balance. Therefore its apparent weight is zero. If the apparent weight of satellite is zero, then all the bodies inside the satellite will feel weightlessness in satellite.



It should be kept in mind that weightlessness does not mean zero gravity. To overcome the problem created due to weightlessness artificial gravity is created.

Critical Velocity:

The minimum velocity required to put a satellite in orbit is called critical velocity. Centripetal acceleration acts on the satellites when it is launched. But the higher speed of satellite enables it to orbit around the earth, centripetal acceleration is given as:

$$a_c = \frac{v^2}{R} \quad \text{but} \quad a_c = g$$

$$\text{So} \quad g = \frac{v^2}{R}$$

$$\text{OR} \quad v = \sqrt{Rg}$$

$$\text{Put } R = 6.4 \times 10^6 \text{ m}, \quad g = 9.8 \text{ m/s}^2$$

$$v = 7.9 \text{ km/s} \approx 8000 \text{ m/s} \approx 8 \text{ km/s}$$



Cannon ball shot parallel to the horizontal surface of the earth

This is the minimum needed velocity that must be given to a satellite to orbit around earth.

Q. 10: Define geo-stationary satellite, derive relation for radius of the orbit of geo-stationary satellite.

Ans. Geo-Stationary Satellite:

The satellite(s) which appear to be stationary if viewed from earth are called geo-stationary satellites.

Time Period of Geo-Stationary Satellites:

These satellites take 24 hours 86400 seconds to complete one rotation around earth. This is the reason that they seem to be stationary.

Mathematically:

Consider a geo-stationary satellite orbiting around earth, the centripetal force is provided by gravitational force i.e.

$$\frac{mv_0^2}{r} = \frac{GmM_e}{r_0^2}$$

$$\text{OR } v_0^2 = \frac{GM_e}{r_0} \text{----- (i)}$$

$$\text{But } v_0 = \frac{S}{T} = \frac{2\pi r_0}{T}$$

Square both sides

$$v_0^2 = \frac{4\pi^2 r_0^2}{T^2} \text{----- (ii)}$$

Compare equations (i) and (ii)

$$\frac{GM_e}{r_0} = \frac{4\pi^2 r_0^2}{T^2}$$

Rearrange the equation

$$r_0^3 = \frac{GMT^2}{4\pi^2}$$

$$\text{OR } r_0 = \left[\frac{GMT^2}{4\pi^2} \right]^{\frac{1}{3}}$$

$$\text{Also } r_0 = \left[\frac{GM}{4\pi^2} \right] (T^2)^{\frac{1}{3}}$$

$$\text{But } \left(\frac{GM}{4\pi^2} \right) \text{ are constants}$$

$$r_0 = \text{constant } T^{\frac{2}{3}}$$

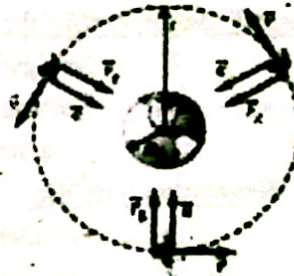
$$\text{OR } r_0 \propto T^{\frac{2}{3}}$$

$$\text{OR } r_0^3 \propto T^2$$

$$\text{OR } T^2 \propto r_0^3$$

This shows that square of time period of a geo-stationary satellite is directly proportional to cube of the radius of the orbit of the geo-stationary satellite.

Q.11: What is artificial satellite? Discuss the purpose of different types of artificial satellites.

Ans. Artificial Satellite:

The satellite is in a circular orbit; its acceleration a is always perpendicular to its velocity v , so its speed v is constant.

The man made satellite which orbit (كرومنا) around earth for different purposes are known as artificial satellites.

Velocity of Artificial Satellites:

The velocity of artificial satellite is given by

$$V = \sqrt{\frac{GM_e}{r}}$$

Where

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$$

M_e = Mass of earth

r = Radius of orbit

Different Types of Artificial Satellites:

- i) Navigation satellite
- ii) Communication satellite
- iii) Weather satellite

i) Navigation Satellite:

These satellites generate positioning information, and provide information regarding geo-spatial positioning and help in time synchronization.

The GPS (Global Positioning System) consists of 24 artificial satellites that orbit at a height of 20,000 km above the earth's surface. This is used to find the exact location and time.

ii) Communication Satellite:

These satellites use artificial satellites to provided communication lines between various regions or points on earth.

These satellites are used for radio, television, phone and internet transmissions. The area or region to which a satellite can transmit is called footprint of satellite.

iii) Weather Satellite:

These satellite use artificial satellite to monitor the weather and climate of earth.

These satellite image (تصوير بنانا) clouds and measure the temperature, pressure rainfall and speed of wind etc. the primary purpose of weather satellites is to forecast weather of the earth. These are low altitude (height) satellites.

MCQ's

1. The angular speed in radians/hours for daily rotation of our earth is?
(A). 2π (B). 4π (C). $\pi/6$ (D). $\pi/12$
2. Linear acceleration $a = r \alpha$ when ' θ '
(A). 0° (B). 180° (C). 360° (D). 90°
3. What is moment of inertia of sphere
(A). MR^2 (B). $1/2 MR^2$ (C). $2/5 MR^2$ (D). $1/2 M^2R$

- (A). Linear momentum (B). Velocity is constant (C). It moves in circular path
(D). Particle moves in straight line
5. A body moving in a circular path with constant speed has
(A). Constant acceleration (B). Constant retardation (C). Variable acceleration
(D). Variable speed & constant velocity
6. Astronauts appear weightless in space because
(A). There is no gravity in space (B). There is no floor pushing upwards
(C). Satellite is freely falling (D). There is no air in space
7. Which is constant for a satellite in orbit?
(A). Velocity (B). K.E (C). Angular momentum (D). Potential energy
8. If the earth suddenly stops rotating the value of 'g' at equator would:
(A). Decrease (B). Remain unchanged (C). Increase (D). Become zero
9. If a solid sphere and solid cylinder of the same mass and density rotate about their own axis, the moment of inertia will be greater for...
(A). Solid sphere (B). Solid cylinder (C). The one that has the largest mass arrives first
(D). The one that has the largest radius arrives first
10. The gravitational force exerted on a astronaut on Earth's surface is 650 N down. When she is in the international space station, the gravitational force on her is.....
(A). Larger than 650 N (B). Exactly the same (C). Smaller
(D). Exactly zero
11. A solid cylinder of mass 'M' and radius 'R' rolls down an incline without slipping. Its moment of inertia about an axis through its center is $\frac{1}{2}MR^2$. At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy?
(A). $\frac{1}{2}$ (B). $\frac{1}{4}$ (C). $\frac{1}{3}$ (D). $\frac{2}{5}$

Answer Keys:

1	D	4	C	7	C	10	C
2	D	5	A	8	C	11	C
3	C	6	C	9	B		

MCOs Solution

1. $\omega = \frac{2\pi}{T}$ Put $T = 24hr$

$$\omega = \frac{2\pi}{24} \text{ rad/hr} = \frac{\pi}{12} \text{ rad/hr}$$

Ans. D. $\frac{\pi}{12} \text{ rad/hr}$

2. $\vec{a} = \vec{\alpha} \times \vec{r}$

$$a = \alpha r \sin \theta$$

If $\theta = 90^\circ$

$$a = \alpha r \sin 90^\circ$$

$$a = \alpha r (1)$$

$$a = \alpha r$$

Ans. D. 90°

3. Ans. C. $\frac{2}{5}MR^2$

4. Ans. C. It moves in a circular path

5. Ans. A. constant acceleration
 6. Ans. C. Satellite is freely falling
 7. Ans. C. Angular momentum
 8. Ans. C. increase

9. **Solution:** For sphere momentum of inertia

$$I_{\text{sphere}} = \frac{2}{5} MR^2 \text{ ----- (1)}$$

$$\text{For cylinder, } I_{\text{cylinder}} = \frac{1}{2} MR^2 \text{ ----- (2)}$$

Dividing equation (2) by equation (1) we get that

$$\frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{\frac{1}{2} MR^2}{\frac{2}{5} MR^2}$$

$$\frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$$

$$\frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{5}{4}$$

$$I_{\text{cylinder}} = \left(\frac{5}{4}\right) I_{\text{sphere}} \text{ ----- (3)}$$

$$\text{As } \frac{5}{4} > 1$$

$$\text{So equation (3)} \Rightarrow I_{\text{cylinder}} > I_{\text{sphere}}$$

Ans. B Solid Cylinder

10. Ans. C. Smaller

Reason:

$$\text{As } F = \frac{GM_e m}{r^2}$$

$$\Rightarrow F \propto \frac{1}{r^2}$$

The distance of international space is very far from earth so the force will be smaller.

11. Find $\frac{(K.E)_{\text{rotational}}}{(K.E)_{\text{total}}} = ?$

$$\text{As } (K.E)_{\text{rotational}} = \frac{1}{2} I \omega^2 \text{ ----- (1)}$$

$$(K.E)_{\text{Total}} = (K.E)_{\text{rotational}} + (K.E)_{\text{translational}}$$

$$(K.E)_{\text{Total}} = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 \quad (\because v = R\omega)$$

$$(K.E)_{\text{Total}} = \frac{1}{2} I \omega^2 + \frac{1}{2} m (R\omega)^2$$

$$(K.E)_{\text{Total}} = \frac{1}{2} I \omega^2 + \frac{1}{2} m R^2 \omega^2$$

$$\left(\because \text{For cylinder} \right. \\ \left. I = \frac{1}{2} MR^2 \right)$$

$$(K.E)_{Total} = \frac{1}{2} I \omega^2 + I \omega^2$$

$$(K.E)_{Total} = I \omega^2 \left(\frac{1}{2} + 1 \right)$$

$$(K.E)_{Total} = I \omega^2 \left(\frac{1+2}{2} \right)$$

$$(K.E)_{Total} = \frac{2}{3} I \omega^2 \text{ ----- (2)}$$

$$\text{Now } \frac{(K.E)_{Total}}{(K.E)_{Total}} = \frac{\frac{1}{2} I \omega^2}{\frac{2}{3} I \omega^2} = \frac{1}{2} + \frac{3}{2} = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$$\text{Ans. C. } \frac{1}{3}$$

Short Questions

Q1: Why is the fly wheel of an engine made heavy in the rim?

Ans. Thesis Statement:

The fly wheel of an engine is made heavy in the rim in order to increase its moment of inertia and ensure (يُثَبِّت) uniform angular motion of the flywheel.

Mathematical Interpretation:

When the fly wheel starts rotation torque is produced in it which is given by:

$$\tau = I \alpha$$

$$\Rightarrow \alpha = \frac{\tau}{I} \text{ ----- (i)}$$

$$(i) \Rightarrow \alpha = \frac{\tau}{m r^2} \text{ ----- (ii)}$$

$$\therefore I = m r^2$$

$I = \text{moment of inertia}$

Theoretical Interpretation:

Since the piston of an engine performs a jerky motion (جَهْشَكَوَالِي حَرَكَت) which is undesirable (نَافِسَد).

From equation (ii) it is clear that angular acceleration (α) and moment of inertia ($I = m r^2$) are inversely related to each other. Which means greater moment of inertia ($m r^2$) smaller will be the angular acceleration.

Since moment of inertia ($I = m r^2$) this means more mass "m" at the rim "r" greater is the moment of inertia ($m r^2$) and smaller is the angular acceleration and the motion will be uniform.



Conclusion:

The conclusion is to reduce the angular acceleration and ensure uniform motion of the flywheel, inertia must be increased which is done by making the flywheel of an engine heavy in the rim.

Q. 2: Why is a barrel "rifled"?

Ans. Thesis Statement:

The barrel of a rifle is rifled to impart spin to the bullet which increases the accuracy and range of bullet.

Rifling the Barrel:

This means the helical/spiral grooves made in the interior of the barrel (بندوق کی نلی).

Explanation:

When a gun fires a bullet, spiral grooves in the barrel produce spin motion in the bullet around its longitudinal axis. Now the bullet possesses two velocities.

- Linear velocity and
- Angular velocity (due to spinning)

Therefore the total kinetic energy of the bullet is given as:

$$K.E_T = K.E_{trans} + K.E_{rotational}$$

$$\Rightarrow K.E_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \text{ ----- (i)}$$

Thus the K.E of the bullet has increased. Both the velocities i.e. linear "v" and angular "ω" are in the same direction which increases the range and accuracy of the bullet.

Conclusion:

Thus it is concluded that the barrel of the rifle is rifled to increase its range and accuracy.

Q. 3: Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on the table? Explain.

Ans. Thesis Statement:

Yes, it is possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on the table on the basis of small and high angular velocities due to moment of inertia.

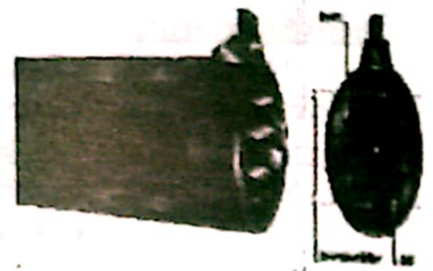
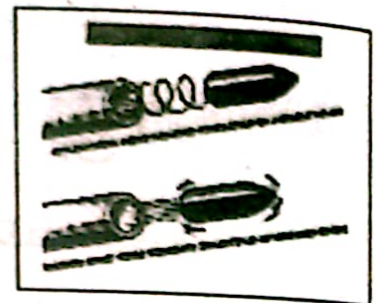
Explanation:

As the angular velocity "ω" of a body is inversely proportional to the moment of inertia (I).

$$\omega = \frac{L}{I} \rightarrow (i) \quad \therefore L = I\omega$$

This shows that greater the moment of inertia smaller will be the angular velocity "ω".

Raw Egg:



The mass of the raw egg will move to the sides, like the rim of a wheel, which will increase its moment of inertia ($I = mr^2$) due to which its " ω " angular velocity will be small and will come to rest quickly.

Hard Boiled Egg:

The mass of the hard egg does not move to the side and therefore has low moment of inertia due to which its angular velocity will be high and will rotate/spin for a longer time as compared to the raw egg.

$$\omega_{\text{raw}} < \omega_{\text{boiled}}$$

The raw egg will stop spinning first and quickly while the boiled egg will continue spinning for a longer time. Thus they can be distinguished from each other.

Conclusion:

Thus it is concluded that it is possible to distinguish a raw egg from a hard boiled one by spinning each on the table.

Q. 4: Why is the acceleration of a body moving uniformly in a circle, directed towards the center?

Ans. Thesis Statement:

The acceleration of a body moving in a circle is due to the centripetal force which is directed towards the center that is why the acceleration of the body is always directed towards the center of the circle.

Explanation:

The velocity of a body moving uniformly in a circle changes its direction at each and every point because of the centripetal force F_c acting on it. The centripetal force is directed towards the center of the circle. Therefore acceleration produced by this force is called centripetal acceleration and is always directed towards the center of the circle.

Mathematical Interpretation:

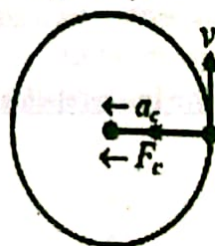
The centripetal force that keeps a body in circular motion is given by,

$$F_c = \frac{-mv^2}{r}$$

$$\Rightarrow m a_c = \frac{-mv^2}{r}$$

$$\therefore F_c = m a_c$$

$$\Rightarrow a_c = \frac{-v^2}{r}$$



This is the centripetal acceleration and the negative sign (-) shows it is directed towards the center.

Q. 5: A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?

Ans. Thesis Statement:

A ball just supported by a string without breaking, if it is set swinging the centrifugal force increases due to which the tension in the string also increases that is why it breaks.

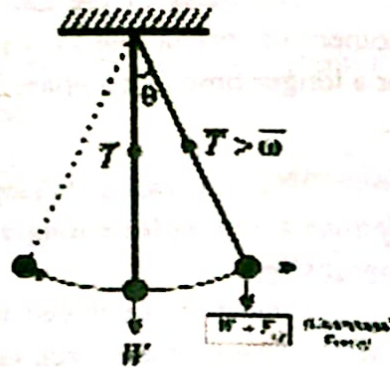
Explanation:

When the ball is just supported by the string the weight of the ball is equal to the tension "T" in the string:

$$\bar{T} = \bar{W}$$

$$T = mg$$

When the ball is set swinging two forces start acting on it. Centripetal force acts towards the center while the centrifugal force " $F_{\text{centrifugal}}$ " acts away from the center due to which the ball tends (کوشش کرنا) to move away from the path. This increases the tension in the string and that is why the string breaks. $\bar{T} > mg$



Conclusion:

Thus it is concluded that the string breaks due to increased tension in the string.

Q. 6: An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.

Ans. Thesis Statement:

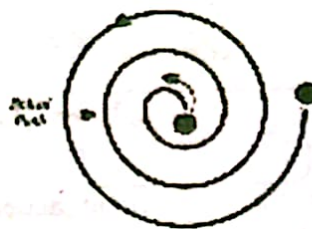
An insect sitting close to the axis of a wheel will start outward spiral motion when the friction between the insect and the wheel is very small.

Explanation:

When the wheel starts rotation, the insect will also rotate. The friction between the insect and the wheel provides the centripetal force to the insect and tends to keep it rotating with the wheel.

On the other hand centrifugal force is also acting on the insect and tends (کوشش کرنا) to make it fly away. As the wheel speeds up, the centrifugal force increases and the insect gradually slips away in a spiral path. Finally the insect will fly away from the wheel.

Figurative Interpretation:



Conclusion:

Thus from the above discussion it is concluded that "the insect will start moving outward in a spiral path".

Q. 7: Explain how many numbers of geo-stationary satellites are required for global coverage of T.V transmission?

Ans. Thesis Statement:

A minimum of three geo-stationary satellites are required for the global coverage of T.V transmission.

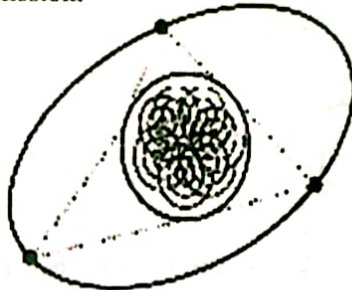
Explanation:

One geo-stationary satellite can cover 120 degree of longitude and there are total 360 degrees of longitude.

Mathematically:

$$\frac{360^\circ}{120^\circ} = 3$$

Thus the minimum of three geo-stationary satellites are required for the global coverage of T.V transmission.



Conclusion:

Thus it can be concluded that a minimum of three geo-stationary satellites are required for the global coverage of T.V transmission.

Q. 8: Explain the significance of moment of inertia in rotatory motion.

Ans. **Thesis Statement:**

The significance of moment of inertia in rotatory motion is the same as that of the mass in translatory motion.

Moment of Inertia:

The property of a rotating body by virtue of which it maintains its state of rest or state of uniform rotatory motion. It is denoted by "I" and is given as: $(I = mr^2)$

"m" is the mass and "r" is the distance of mass from the center of rotation.

Significance:

The moment of inertia "I" in rotatory motion is analogous to mass (measure of inertia) in translatory motion.

It resists any change in the rotatory motion of a body. The angular acceleration is inversely proportional to the moment of inertia. i.e.

$$\alpha = \frac{\tau}{I}$$

$$\alpha = \frac{\tau}{mr^2}$$

Thus by changing moment of inertia angular acceleration can be changed.

Conclusion:

Thus it is concluded that the significance of moment of inertia in rotatory motion is the same as that of mass in translatory motion.

Q. 9: Why does the coasting rotating system slow down as water drops into the beaker?

Ans. **Thesis Statement:**

The coasting rotating system slows down as water drops into the beaker due to increase in moment of inertia which in turn decreases the angular velocity of the system.

Explanation:

From the law of conservation of angular momentum

$$L = I\omega$$

$$\Rightarrow \omega = \frac{L}{I}$$

$$\Rightarrow \omega \propto \frac{1}{I} \rightarrow (i)$$

Also

$$I = mr^2$$

As more water is added to the system its mass increases due to which the moment of inertia is increased and the angular velocity decreases and thus the coasting rotating system slows down.

Conclusion:

Thus the conclusion is that the coasting rotating system slows down when water is added to it due to increased moment of inertia. ($I = mr^2$)

Q. 10: A body will be weightless when the elevator falls down just like a free falling body. Explain.

Ans. Thesis Statement:

A body will be weightless when the elevator falls down just like a freely falling body because the tension in the string of the physical balance is zero i.e. the apparent weight is zero.

Explanation:

Consider an elevator is moving downward with acceleration "a". The body inside the elevator also move the same acceleration "a". Therefore the net force acting on the body of mass "m" is given by:

$$F_{net} = W - T$$

$$\Rightarrow T = W - F_{net}$$

$$\Rightarrow T = mg - ma \rightarrow (i)$$

$$W = mg, \text{ actual weight of the body}$$

$$T, \text{ tension in the string of spring balance}$$

T is the tension in the string that reads the apparent weight of the body, which is less than the real weight of the body "mg" by an amount "ma".

If the elevator is falling freely under gravity, its acceleration is $a = g$, putting in equation (i).

$$(i) \Rightarrow T = mg - mg$$

$$T = 0 \rightarrow (ii)$$

This shows that the apparent weight of the body is zero, and the body is weightless when the elevator falls freely.

Key Note:

The actual weight of the body can never be zero it is given by ($W = mg$) only the apparent weight i.e. the tension in the string of physical balance can be zero, as in the above case.

Conclusion:

Thus the conclusion is that a body will be weightless when the elevator fall down just like freely falling body because it's apparent weight becomes zero.

Q. 11: When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel, why? Explain.

Ans. **Thesis Statement:**

When a tractor moves with a uniform velocity, its heavy wheel must rotate slowly than the lighter wheel so that the linear displacement covered by both the wheels be equal.

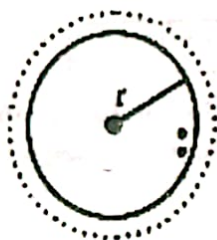
Explanation:

Consider "r" is the radius of the lighter wheel. The linear distance covered by this wheel is equal to its circumference i.e.

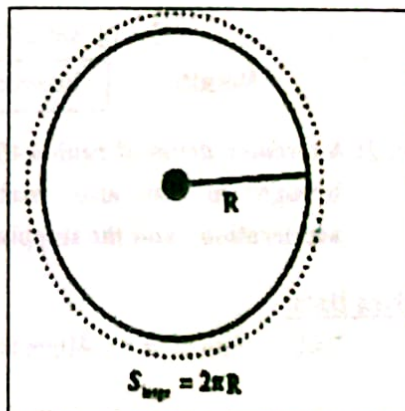
$$S_{\text{small}} = 2\pi r \rightarrow (i)$$

When as the radius of heavier wheel is "R" so, distance covered by this wheel is,

$$S_{\text{large}} = 2\pi R \rightarrow (ii)$$



$$S = 2\pi r$$



$$S_{\text{large}} = 2\pi R$$

From (i) and (ii) it is clear that circumference of the heavy wheel is large as compared to the lighter wheel. Therefore in order to cover same distance on the ground the heavier wheel will rotate slowly then the lighter wheel.

Secondly for equal linear distance the heavier wheel will rotate slowly due to the large radius according to the equation.

$$\omega = \frac{v}{r} \Rightarrow \omega \propto \frac{1}{r}$$

Numerical Problems

Problem. 1: If the plate microwave oven has a radius of 0.15 m and rotates at 6.0 rev/min, calculate the total distance traveled by the fly during a 2.0 min cooking period.

Given Data:

Radius of the plate microwave oven = $r = 0.15\text{m}$

$$\text{Angular velocity} = \omega = 6 \text{ rev/min} = 6 \frac{2\pi}{60} \text{ rad/sec} \\ = 0.628 \text{ rad/sec}$$

$$\text{Time period} = T = 2 \text{ min} = 2 \times 60 \text{ sec} = 120 \text{ sec}$$

Required Data:

$$\text{Distance travelled} = S = ?$$

Solution:

We know that

$$S = r\theta \text{ ----- (1)}$$

We also know that

$$\omega = \frac{\theta}{T}$$

$$\Rightarrow \theta = \omega T \text{ ----- (2)}$$

Putting equation (2) in equation (1) we get that

$$S = r\omega T \text{ ----- (3)}$$

Putting values in equation (3) we get,

$$S = (0.15)(0.628)(120)$$

$$S = 11.3 \text{ m} = 11 \text{ m}$$

Result:

$$\text{Distance travelled} = S = 11 \text{ m}$$

Problem. 2: A circular drum of radius 40 cm is initially rotating at 400 rev/min. It is brought to stop after making 50 revolutions. What is the angular acceleration and the stopping time?

Given Data:

$$\text{Radius of drum} = r = 40 \text{ cm} = 0.4 \text{ m}$$

$$\text{Initial angular velocity} = \omega_i = 400 \frac{\text{rev}}{\text{min}} = 400 \times \frac{2\pi}{60} \frac{\text{rad}}{\text{min}} \\ = 41.866 \text{ rad/sec}$$

$$\text{Angular displacement} = \theta = 50 \text{ revolution} = 50(2\pi \text{ rad}) = 100\pi \text{ rad}$$

$$\theta = 100(3.14) \text{ rad} = 314 \text{ rad}$$

$$\text{Final angular velocity} = \omega_f = 0 \text{ rad/sec}$$

Required Data:

a) Angular acceleration = $\alpha = ?$

b) Stopping time = $t = ?$

Solution (a):

As we know that

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \text{ ----- (1)}$$

Putting values in equation (1) we get

$$2\alpha(314) = 0^2 - (41.866)^2$$

$$628\alpha = -1752.76$$

$$\alpha = \frac{-1752.76}{628}$$

Result:

$$\alpha = -2.79 \text{ rad/sec}^2$$

Solution (b):

We know that

$$\omega_f = \omega_i + \alpha t \text{ ----- (2)}$$

Putting values in equation (2)

$$0 = (41.866) + (-2.79)t$$

$$2.79t = 41.866$$

$$t = \frac{41.866}{2.79}$$

$$t = 15 \text{ sec}$$

Result:

$$\text{Stopping time} = t = 15 \text{ sec}$$

Problem. 3: A string 1 m long is used to whirl a 100 g stone in a horizontal circle at a speed of 2 ms^{-1} . Find the tension in the string.

Given Data:

Length of string = $l = 1 \text{ m}$

Mass of stone = $m = 100 \text{ g} = 0.1 \text{ kg}$

Speed of stone = $v = 2 \text{ m/sec}$

Required Data:

Tension in the string = $T = ?$

Solution:

Here in this case the tension in the string must be equal to centripetal force.

$$\text{i.e. } T = F_c = \frac{mv^2}{r} \text{ ----- (1)}$$

Putting values in equation (1)

$$T = \frac{(0.1)(2)^2}{1} = \frac{0.1 \times 4}{1} = 0.4 \text{ N} \quad (N = r = 1 \text{ m})$$

Result:

$$\text{Tension in the string} = T = 0.4 \text{ N}$$

Problem. 4: The moon revolves around the earth in almost a circle of radius 382400 km in 27.3 days. What is the centripetal acceleration?

Given Data:

Radius of circle = $r = 382400 \text{ km}$

$$= 382400 \times 10^3 \text{ m}$$

$$= 3824 \times 10^5 \text{ m}$$

Required: Centripetal acceleration = $a_c = ?$

Solution: We know that

$$a_c = r\omega^2$$

$$a_c = r\left(\frac{2\pi}{T}\right)^2$$

$$a_c = \frac{4\pi^2 r}{T^2} \text{-----(1)}$$

Putting values in equation (1)

$$a_c = \frac{4(3.14)^2 (38200 \times 10^3)}{(235820)^2}$$

Result:

$$a_c = 0.00271 \text{ m/sec}^2$$

Problem. 5: A modern F1 car can accelerate from 0 to 62 mile/h (100 km/h) in 2.50 s.
What is the angular acceleration of its 170 mm-radius wheels?

Given Data:

Initial velocity = $v_i = 0 \text{ mile/hr} = 0 \text{ m/sec}$

Final velocity = $v_f = 62 \text{ mile/hr} = 100 \text{ km/hr}$

$$= 100 \times \frac{1000}{3600} \text{ m/sec}$$

$$= 27.8 \text{ m/sec}$$

Time taken = $t = 2.5 \text{ sec}$

Radius of wheel = $r = 170 \text{ mm} = 170 \times 10^{-3} \text{ m} = 0.170 \text{ m}$

Required:

Angular acceleration = $\alpha = ?$

Solution:

We know that

$$\omega_f = \omega_i + \alpha t$$

$\Rightarrow$

$$\frac{v_f}{r} = \frac{v_i}{r} + \alpha t$$

$$\frac{v_f}{r} - \frac{v_i}{r} = \alpha t$$

$$\alpha t = \frac{v_f}{r} - \frac{v_i}{r}$$

$$\alpha t = \frac{1}{r} - \frac{v_i}{r}$$

$$\alpha t = \frac{1}{r} (v_f - v_i)$$

$$\alpha = \frac{1}{rt} (v_f - v_i) \text{-----(1)}$$

$$v = r\omega$$

$$\omega = \frac{v}{r}$$

$$\omega_i = \frac{v_i}{r}$$

$$\omega_f = \frac{v_f}{r}$$

Putting values in equation (1)

$$\alpha = \frac{1}{(0.170)(2.5)} [27.8 - 0]$$

$$\alpha = \frac{27.8}{0.425} = 65.4 \text{ rad/sec}^2$$

Result:

$$\alpha = 65.4 \text{ rad/sec}^2$$

Problem. 6: An electric motor is running at $1800 \text{ rev/min}^{-1}$. It comes to rest in 20 s . If the angular acceleration is uniform, find number of revolutions it made before stopping.

Given Data:

Initial angular momentum = $\omega_i = 180 \text{ rev/min}$

$$= \frac{180}{60} \text{ rev/sec} = 30 \text{ rev/sec}$$

Final angular momentum = $\omega_f = 0 \text{ rev/sec}$

Time taken = $t = 20 \text{ sec}$

Required Data:

Number of revolution = $\theta = ?$

Solution:

As we know that

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\theta = \omega_i t + \frac{1}{2} \left(\frac{\omega_f - \omega_i}{t} \right) t^2 \quad \left[\alpha = \frac{\omega_f - \omega_i}{t} \right]$$

$$\theta = \omega_i t + \frac{1}{2} (\omega_f - \omega_i) t \text{ ----- (1)}$$

Putting values in equation (1)

$$\theta = (30)(20) + \frac{1}{2} (0 - 30)(20)$$

$$\theta = 600 + \frac{1}{2} (-600)$$

$$\theta = 300 \text{ rev}$$

Result:

$$\text{Number of revolution} = \theta = 300 \text{ rev}$$

Problem. 7: What is the moment of inertia of a 100 kg sphere whose radius is 50 cm ?

Given Data:

Mass of sphere = $m = 100 \text{ kg}$

Radius of sphere = $R = 50 \text{ cm} = 0.5 \text{ m}$

Required Data:

Moment of inertia of sphere = $I_{\text{sphere}} = ?$

Solution:

As we know that moment of inertia of sphere is given by

$$I_{\text{sphere}} = \frac{2}{5} M R^2 \text{ ----- (1)}$$

$$I_{\text{sphere}} = \frac{2}{5}(100)(0.5)^2$$

$$I_{\text{sphere}} = 2(20)(0.5)^2$$

Result:

$$I_{\text{sphere}} = 10 \text{ kgm}^2$$

Problem. 8: A rope is wrapped several times around a cylinder of radius 0.2 m and mass 30 kg. What is the angular acceleration of the cylinder if the tension in the rope is 40 N and it turns without friction?

Given Data:

Radius of cylinder = $r = 0.2 \text{ m}$

Mass of cylinder = $m = 30 \text{ kg}$

Tension in the rope = $T = F = 40 \text{ N}$

Required Data:

Solution:

As we know that

$$\tau = I\alpha \text{ ----- (1)}$$

We also know that

$$\tau = rF \text{ ----- (2)}$$

Comparing equation (1) & equation (2)

$$I\alpha = rF$$

$$\alpha = \frac{rF}{I}$$

$$\alpha = \frac{rF}{\frac{1}{2}mr^2} = \frac{2rF}{mr^2} \text{ ----- (3)}$$

Putting values in equation (3)

$$\alpha = \frac{(2)(0.2)(40)}{(30)(0.2)^2} = 13.3 \text{ rad / sec}^2$$

Result:

$$\alpha = 13.3 \text{ rad / sec}^2$$

Momentum of inertia of cylinder is

$$I = \frac{1}{2}mr^2$$

Problem. 9: What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15 m if it rolls across a level surface with a speed of 2 ms^{-1} ?

Given Data:

Mass of ball = $m = 5 \text{ kg}$

Diameter of ball = $d = 15 \text{ m}$

Radius of ball = $R = \frac{d}{2} = \frac{15}{2} = 7.5 \text{ m}$

Speed of ball = $V = 2 \text{ m / sec}$

Required Data:

K.E of rolling ball = $K.E = ?$

Solution:

The rolling ball will have both linear and rotational K.E. So the total K.E will be:

$$(K.E)_{Total} = (K.E)_{linear} + (K.E)_{rot}$$

$$\Rightarrow (K.E)_{Total} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$\Rightarrow (K.E)_{Total} = \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{2}{5} m R^2 \right) \left(\frac{v}{R} \right)^2$$

$$\Rightarrow (K.E)_{Total} = \frac{1}{2} m v^2 + \frac{1}{5} m v^2 \text{ ----- (1)}$$

For solid sphere/ball

$$I = \frac{2}{5} M R^2$$

$$\text{Also } V = R\omega$$

$$\omega = \frac{v}{R}$$

Putting values in equation (1) we get

$$(K.E)_{Total} = \frac{1}{2} (5) (2)^2 + \frac{1}{5} (2)^2$$

$$(K.E)_{Total} = \frac{20}{2} + \frac{20}{5} = 10 + 4 = 14 \text{ Joule}$$

Result:

$$\text{Total K.E of rolling ball} = (K.E)_{Total} = 14 \text{ J}$$

Problem. 10: A cylinder of 50 cm diameter at the top of an incline 29.4 cm high and 10 m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction.

Given Data:

$$\text{Diameter of cylinder} = D = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{Radius of cylinder} = R = \frac{D}{2} = \frac{0.5}{2} = 0.25 \text{ m}$$

$$\text{Height} = h = 29.4 \text{ cm} = 0.294 \text{ m}$$

Required:

$$\text{Linear speed} = V = ?$$

$$\text{Angular speed} = \omega = ?$$

Solution (a):

As the cylinder is rolling from top to bottom. So by law of conservation of energy:

$$\text{Loss in PE} = \text{Gain in K.E}$$

$$mgh = (K.E)_{linear} + (K.E)_{rot}$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{1}{2} m r^2 \right) \left(\frac{v}{r} \right)^2$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{4} m v^2$$

$$mgh = m \left(\frac{v^2}{2} + \frac{v^2}{4} \right)$$

$$gh = \frac{2v^2 + v^2}{4} = \frac{3v^2}{4}$$

$$\Rightarrow gh = \frac{3v^2}{4} \Rightarrow v = \sqrt{\frac{4}{3} gh}$$

For cylinder

$$I = \frac{1}{2} m R^2$$

$$\& v = r\omega$$

$$\omega = \frac{v}{r}$$

Putting values

$$\Rightarrow v = \sqrt{\frac{4(9.8)(0.294)}{3}} = 1.96 \text{ m/sec}$$

Result:

$$\Rightarrow v = 1.96 \text{ m/sec}$$

Solution (b):Now to find " ω "

$$\text{As } v = R\omega$$

$$\omega = \frac{v}{R}$$

Result:

$$\omega = \frac{1.96}{0.25} = \omega = 7.84 \text{ rad/sec}$$

Problem. 11: A disc without slipping rolls down a hill of vertical height 1000 cm if the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

Given Data:

$$\text{Height hill} = h = 1000 \text{ cm} = 10 \text{ m}$$

Required Data:

$$\text{Speed at the bottom} = v = ?$$

Solution:

As the disc slipping rolls from top to bottom. So by the law of conservation of energy:

$$\text{Loss in PE} = \text{Gain in K.E}$$

$$mgh = (K.E)_{\text{linear}} + (K.E)_{\text{rot}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}m \cancel{r^2} \frac{v^2}{\cancel{r^2}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\cancel{m}gh = \cancel{m}\left(\frac{v^2}{2} + \frac{v^2}{4}\right)$$

$$gh = \frac{2v^2 + v^2}{4} = \frac{3v^2}{4}$$

$$\Rightarrow v = \sqrt{\frac{4}{3}gh}$$

$$\Rightarrow v = \sqrt{\frac{4}{3}(9.8)(10)} = 11.4 \text{ m/s}$$

Result:

$$\Rightarrow v = 11.4 \text{ m/sec}$$

For cylinder

$$I = \frac{1}{2}mR^2$$

$$\& v = r\omega$$

$$\omega = \frac{v}{r}$$

Problem. 12: A motor car is traveling at a speed of 30 ms^{-1} . If its wheel has a diameter of 1.5 m , find its angular speed in rads^{-1} and revs^{-1} .

Given Data:

Speed of car = $v = 30 \text{ m/sec}$

Diameter of wheel = $D = 1.5 \text{ m}$

Radius of wheel = $R = \frac{D}{2} = \frac{1.5}{2} = 0.75 \text{ m}$

Required:

a) Angular speed = $\omega_{(\text{rad/sec})} = ?$

b) Angular speed = $\omega_{(\text{rev/sec})} = ?$

Solution (a):

As we know that $v = R\omega$

$\Rightarrow \omega = \frac{v}{R}$

Putting values

$\omega = \frac{30}{0.75} = 40 \text{ rad/sec}$

Result:

$\omega = 40 \text{ rad/sec}$

Solution (b):

$\omega = 40 \text{ rad/sec}$

$\omega = 40 \frac{1}{2\pi} \text{ rev/sec}$

$\omega = \frac{40}{2(3.14)} = 6.37 \text{ rev/sec}$

$2\pi \text{ rad} = 1 \text{ revolution}$

$1 \text{ rad} = \frac{1}{2\pi} \text{ revolution}$

Result:

$\omega = 6.37 \text{ rev/sec}$

Assignments

Assignment 5.1: A rotating pulley completes 12 revolutions in 4 s. Determine the average angular velocity in rev/s and in rad/s .

Given Data:

Total angular displacement = $\theta = 12 \text{ rev}$

Total time = $t = 4 \text{ sec}$

Required Data:

a) Average angular velocity in $\text{rev/sec} = ?$

b) Average angular velocity in $\text{rev/min} = ?$

c) Average angular velocity in $\text{rad/sec} = ?$

Solution (a):

As average angular velocity is given by

$\omega_{\text{ave}} = \frac{\text{Total angular displacement}}{\text{Total time taken}}$

$$\omega_{ave} = \frac{12 \text{ rev}}{4 \text{ sec}} = 3 \text{ rev/sec}$$

$$\omega_{ave} = 3 \text{ rev/sec} \rightarrow (1)$$

Solution (b):

As

$$1 \text{ minute} = 60 \text{ sec}$$

$$1 \text{ sec} = \frac{1}{60} \text{ min}$$

$$1 \text{ sec} = 0.0167 \text{ min} \quad \text{Put in (1)}$$

$$\omega_{ave} = \frac{3 \text{ rev}}{0.0167 \text{ min}} = 180 \text{ rev/min}$$

Result:

$$\omega_{ave} = 180 \text{ rev/min}$$

Solution (c):

As

$$1 \text{ rev} = 2\pi \text{ rad}$$

Put in equation (1) we get

$$\omega_{ave} = \frac{3(2\pi \text{ rad})}{\text{sec}}$$

$$\omega_{ave} = 6\pi \text{ rad/sec}$$

$$\omega_{ave} = 6(3.14) \text{ rad/sec}$$

Result:

$$\omega_{ave} = 18.8 \text{ rad/sec}$$

Assignment 5.2: An airplane dives along a curved path of radius 'R' and velocity $\bar{v}$. The centripetal acceleration is 10 ms^{-2} . If both the velocity and radius are doubled, what will be the new acceleration?

Given Data:

$$\text{Centripetal acceleration} = a_c = 10 \text{ m/sec}^2$$

$$\text{If } R' = 2R$$

$$v' = 2v$$

Required Data:

$$\text{New acceleration} = a'_c = ?$$

Solution:

As we know that

$$a_c = \frac{v^2}{R}$$

For new acceleration

$$a'_c = \frac{v'^2}{R'}$$

$$\text{Put } v' = 2v \text{ and } R' = 2R$$

$$a'_c = \frac{(2v)^2}{(2R)}$$

$$a'_c = 4 \frac{v^2}{2R}$$

$$a'_c = 2 \left(\frac{v^2}{R} \right)$$

$$a'_c = 2a_c$$

$$\text{Put value of } a_c = 10 \text{ m/sec}^2$$

$$a'_c = 2(10) \text{ m/sec}^2$$

$$a'_c = 20 \text{ m/sec}^2$$

Result:

Assignment 5.3:

At what speed (in km/h) is a bank angle of 45° required for an aeroplane to turn on a radius of 60 m?

Given Data:

$$\text{Bank angle} = \theta = 45^\circ$$

$$\text{Radius} = r = 60 \text{ m}$$

Required Data:

$$\text{Speed of airplane} = V = ?$$

Solution:

$$\text{As we know that } \tan \theta = \frac{v^2}{rg}$$

$$\Rightarrow v^2 = rg \tan \theta$$

$$v = \sqrt{rg \tan \theta} \text{ ----- (1)}$$

Putting values in equation (1) we get

$$v = \sqrt{(60)(9.8) \tan 45^\circ}$$

$$v = \sqrt{588(1)} = 24.3 \text{ m/sec}$$

$$v = 24.3 \text{ m/sec}$$

OR

$$v = 24.3 \frac{3600}{1000} \text{ km/hr}$$

$$v = 87.3 \text{ km/hr}$$

Result:

Assignment 5.4:

A cord is wrapped around the rim of a cylinder that has a mass of 10 kg and a radius of 30 cm. If the rope is pulled with a force of 60 N, what will be the angular acceleration of the cylinder?

Given Data:

$$\text{Mass} = m = 10 \text{ kg}$$

$$\text{Radius} = r = 30 \text{ cm} = 0.3 \text{ m}$$

$$\text{Force} = F = 60 \text{ N}$$

Required:

$$\text{Angular acceleration} = \alpha = ?$$

Solution:

As we know that $\tau = I\alpha$ ----- (1)

We also know that $\tau = rF$ ----- (2)

Comparing equation (1) & equation (2) we get

$$I\alpha = rF$$

$$\alpha = \frac{rF}{I} \text{ ----- (3)}$$

Moment of inertia for cylinder

$$I = \frac{1}{2}mr^2 \quad \text{Put in equation (3)}$$

$$\alpha = \frac{rF}{\frac{1}{2}mr^2} = \frac{2rF}{mr^2}$$

$$\alpha = \frac{2rF}{mr^2} \text{ ----- (4)}$$

Putting values in equation (4) we have

$$\alpha = \frac{2(0.3)(60)}{(10)(0.3)^2}$$

Result:

$$\alpha = 40 \text{ rad/sec}^2$$

Assignment 5.5:

A belt is wrapped around the edge of a pulley that is 40 cm in diameter. The pulley rotates with a constant angular acceleration of 3.50 rad/s^2 . At $t = 0$, the rotational speed is 2 rad/s . What is the angular displacement and angular velocity of the pulley 2 s later?

Given Data:

Diameter of pulley $= d = 40 \text{ cm} = 0.4 \text{ m}$

Radius of pulley $= r = 20 \text{ cm} = 0.2 \text{ m}$

Angular acceleration $= \alpha = 3.5 \text{ rad/sec}^2$

Initial angular velocity $= \omega_i = 2 \text{ rad/sec}$

Time $= t = 2 \text{ sec}$

Required Data:

a) Final angular velocity $= \omega_f = ?$

b) Angular displacement $= \theta = ?$

Solution (a):

As know that $\omega_f = \omega_i + \alpha t$ ----- (1)

Putting values in equation (1)

$$\omega_f = 2 + (3.5)(2)$$

$$\omega_f = 2 + 7 = 9 \text{ rad/sec}$$

Result:

$$\omega_f = 9 \text{ rad/sec}$$

Solution (b):

We also know that $\theta = \omega t + \frac{1}{2} \alpha t^2$ ----- (2)

Putting values in equation (2)

$$\theta = (2)(2) + \frac{1}{2}(3.5)(2)^2$$

$$\theta = 4 + 7 = 11 \text{ rad}$$

$$\theta = 11 \text{ rad}$$

Result:

Assignment 5.6:

A DVD disc has a radius of 0.0600 m, and a mass of 0.0200 kg. The moment of inertia of a solid disc is $I = \frac{1}{2} MR^2$, where m is mass of the disc, and 'R' is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of 160.0 rads^{-1} . What is the angular momentum of this disc?

Given Data:

Radius of disc = $r = 0.0600 \text{ m}$

Mass of disc = $m = 0.0200 \text{ kg}$

Angular velocity = $\omega = 160 \text{ rad/sec}$

Required Data:

Angular momentum of disc = $L = ?$

Solution:

As we know that

$$L = I \omega$$

$$L = \left(\frac{1}{2} m r^2 \right) \omega$$

$$L = \frac{1}{2} m r^2 \omega$$
 ----- (1)

Putting values in equation (1) we get that

$$L = \frac{1}{2} (0.0200) (0.0600)^2 (160)$$

Result:

$$L = 0.00576 \text{ kgm}^2 / \text{sec}$$

Assignment 5.7:

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2 , what will be the reading of the spring balance?

Given Data:

Real weight = $W = 49 \text{ N}$

Downward acceleration = $a = 5 \text{ m/sec}^2$

Required Data:

Apparent weight = $T = ?$

Solution:

In case of downward accelerated motion the apparent weight of the bag is given by:

$$T = W - ma \text{ ----- (1)}$$

$$T = W - \frac{W}{g} a \text{ ----- (2)}$$

Putting values in equation (2) we get that

$$T = 49 - \frac{49}{9.8}(5)$$

$$T = 49 - (5)(5)$$

$$T = 49 - 25$$

Result:

$$T = 24N$$

$$W = mg$$

$$m = \frac{W}{g}$$

Unit # 6

Fluid Dynamics**Exercise Long Questions Answers**

Q. 1: What is viscous drag? State and explain Stoke's law.

Ans. **Viscous Drag:**

It is a resistive or opposing force acting on any object due to the relative motion of the object with respect to the medium (fluid).

OR

The retarding force experienced (محسوس کرنا) by an object moving through a fluid.

Explanation:

When an object moves through a medium the moving object exerts an action force on the fluid to push it out of the path. The medium exerts a reaction force on the object equal in magnitude to the action of object but opposite in direction. This force opposes the motion of the object through the medium. This force is called drag force or viscous drag.

Examples:

- i) When a person puts his hand out of the window of a moving car, he experiences a considerable amount of air drag force on his hand.
- ii) A person riding over a bike experiences an appreciable amount of air drag force. The drag force increases as the speed of bike increases.
- iii) Swimmers and skydivers manipulate viscous drag by changing their effective size and orientation by bending, twisting and stretching.
- iv) To reduce the viscous drag on moving vehicles, the vehicles are specially designed.



Coefficient of
viscosity ' η '
of medium

Factors Which Affect Viscous Drag:

These are the following.

- Size, shape and orientation of object
- Viscosity of medium
- Speed of object relative to the fluid

The size, shape orientation of object, viscosity of medium and relative speed of object all are directly related with viscous drag.

Nature:

It is a vector, always points opposite to the direction of motion of object.

Unit:

Its unit is that of force, which is newton.

Dimensions:

Its dimensions are that of force, which are $[MLT^{-2}]$.

Stoke's Law:

This law presented by Sir George. G. Stokes in 1851 states, that "the drag force acting on a spherical object is directly proportional to the viscosity " η " of medium, radius " r " and velocity " v " of the object".

Mathematically:

It can expressed as:

$$F_D \propto \eta$$

$$F_D \propto r$$

$$F_D \propto v$$

Combine above relations

$$F_D \propto \eta \propto v$$

OR $F_D = k \eta r v$

where $k = 6\pi$ for spherical object

So $F_D = 6\pi\eta r v$

This equation is called Stoke's Law.

Applications of Stoke's Law:

These are the following:

- Calculation of drag force
- Calculation of viscosity of fluid
- To study the flow of water in dams

Q. 2: What is terminal velocity? Derive mathematical relation for terminal velocity by using Stoke's law.

Ans. Terminal Velocity:

When the drag force becomes equal to the weight of the body, the body then moves with constant (uniform) velocity, which is called terminal velocity.

Explanation:

When an object for instance a rain drop falls it is acted upon by air drag. The direction of air drag is opposite to the weight of rain drop. As the rain drop falls its velocity increases which causes an increase in air drag. At some point through its fall the drag force becomes equal to the weight of the body, at this moment the rain drop falls with uniform velocity with no acceleration, this velocity is called terminal velocity.

Nature:

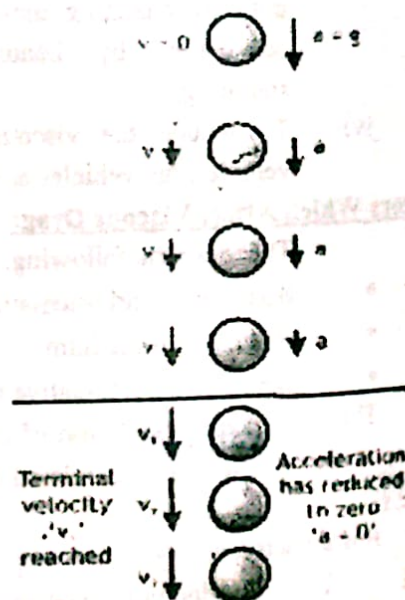
It is a vector, and it always point in the direction of motion of object.

Unit:

Since its velocity, its unit is m/s.

Dimensions

Since its velocity its dimensions are that of velocity i.e. $[LT^{-1}]$.



Terminal Velocity by use of Stoke's Law:
Consider an object (rain drop) falls through air, two forces act on rain drop, the weight of rain drop which is directed downward, and drag force which is directed upward. As the object falls its velocity increases due to gravity which causes an increase in the drag force. When the velocity reaches a certain maximum value the drag force becomes equal to the weight of the object i.e.

$F_D = \text{weight of object}$

$$F_D = mg \text{ ----- (i)}$$

But according to Stoke's Law

$$F_D = 6\pi\eta rv$$

Put in equation (i)

$$6\pi\eta rv_T = mg \text{ ----- (ii)}$$

The object will fall with uniform velocity v_T .

$$(ii) \Rightarrow 6\pi\eta rv_T = mg$$

$$\text{OR } v_T = \frac{mg}{6\pi\eta r} \text{ ----- (iii)}$$

$$\text{But } \rho = \frac{m}{v} \text{ OR } m = \rho v \text{ but } v = \frac{4}{3}\pi r^3$$

$$\Rightarrow m = \rho \left(\frac{4}{3}\pi r^3 \right) \text{ Put in (iii)}$$

$$(iii) \Rightarrow v_T = \frac{\left(\rho \frac{4}{3}\pi r^3 \right) g}{6\pi\eta r}$$

$$\text{OR } v_T = \frac{2}{9} \frac{\rho r^2 g}{\eta}$$

$$\text{OR } v_T = \frac{2\rho g r^2}{9\eta}$$

Terminal velocity V_T is inversely proportional to viscosity of medium η . This physically means that more viscous the fluid, smaller will be the value of terminal velocity for that medium (fluid).

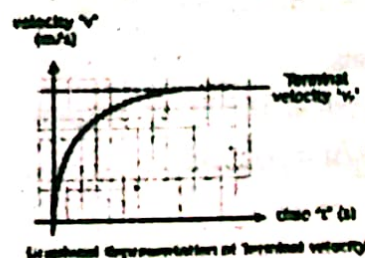
Factors Affecting Terminal Velocity:

- i) Size, shape and orientation of object
- ii) Co-efficient of viscosity of medium

Graphical Explanation of Terminal Velocity:

If the time taken by the object reaching terminal velocity is plotted against velocity the following curve is obtained.

The maximum velocity is represented by v_T (Terminal Velocity).



Q. 3: Derive mathematically the equation of continuity and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?

Ans. Equation of Continuity:

This equation states that for an ideal fluid in a steady flow. The product of cross-sectional area and speed of the fluid at any point along the pipe is constant. i.e.

$$Av = \text{constant}$$

$$\text{OR } A_1v_1 = A_2v_2 = A_3v_3 = \dots = A_nv_n$$

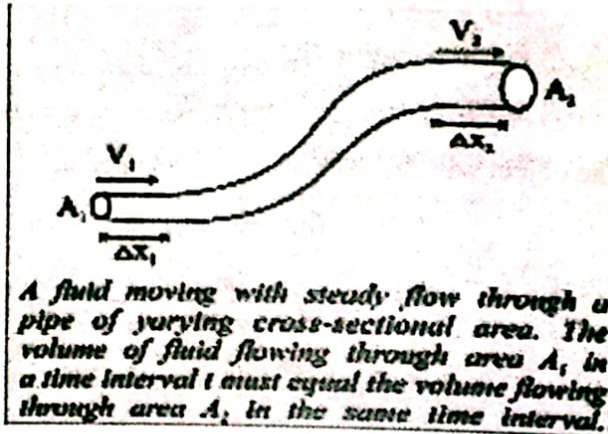
Where v = speed of fluid

A = cross-sectional area of pipe

Mathematical Derivation:

Consider an ideal fluid flowing through a pipe of non-uniform cross-sectional area as shown in figure.

If there are no sink or source of fluid in the pipe, then for ideal fluid the mass Δm entering the pipe must be equal to the mass outgoing from the pipe, i.e.



$$\Delta m_1 = \Delta m_2 = \Delta m \text{-----(i)}$$

By definition of density

$$\rho = \frac{\Delta m}{\Delta V}$$

$$\text{OR } \Delta m = \rho \Delta V \text{-----(ii)}$$

$$\text{But } \Delta V = A \Delta x \text{-----(iii)}$$

$$\text{And } \Delta x = V \Delta t \text{-----(iv)} \quad \text{As } (S = vt)$$

Put $\Delta x = V \Delta t$ in (iii) one get that

$$\Delta V = A V \Delta t$$

Put this value in equation (ii)

$$\Delta m = \rho A V \Delta t$$

For cross-sectional area A_1 it can be expressed as:

$$\Delta m_1 = \rho A_1 V_1 \Delta t \text{----- (v)}$$

Similarly for cross-sectional area A_2

$$\Delta m_2 = \rho A_2 V_2 \Delta t \text{----- (vi)}$$

NOTE: Δt and ρ are constants

Put equation (v) and (vi) in (i)

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$\Rightarrow A_1 v_1 = A_2 v_2$$

If "n" number of cross-sectional areas are considered then

$$A_1 v_1 = A_2 v_2 = A_3 v_3 = \dots \dots \dots A_n v_n$$

This is called continuity equation.

Generalized Form of Continuity Equation:

The generalized form of continuity equation is given as,

$$Av = \text{constant}$$

This is the generalized form of continuity equation.

Relating continuity Equation to the Time Rate of Volume Flow:

According to continuity equation

$$Av = \text{constant}$$

In which

A = cross-sectional area of pipe

v = speed or velocity of fluid

According to definition of velocity/speed

$$v = \frac{\Delta S}{\Delta t}$$

But in present case $S = \Delta x$

$$\text{So } v = \frac{\Delta x}{\Delta t}$$

Put this in continuity equation

$$A \left(\frac{\Delta x}{\Delta t} \right) = \text{constant}$$

$$\text{OR } \left(\frac{A \Delta x}{\Delta t} \right) = \text{constant}$$

But $A \Delta x = \text{Volume of fluid flowing}$

So $A \Delta x = \Delta V$ hence

$$\frac{\Delta V}{\Delta t} = \text{constant}$$

This shows that the time rate of change of volume of fluid is constant.

Conservation of Mass and Continuity Equation:

Continuity equation in terms of time rate of volume of fluid is given by:

$$\frac{\Delta V}{\Delta t} = \text{constant}$$

$$\text{OR } \frac{\Delta V_1}{\Delta t} = \frac{\Delta V_2}{\Delta t} = \frac{\Delta V_3}{\Delta t} = \dots \dots \dots = \frac{\Delta V_n}{\Delta t} \dots \dots \dots (i)$$

By definition of volume $V = \frac{m}{\rho}$

$$\Delta V = \frac{\Delta m}{\rho}$$

Put in equation (i)

$$\frac{\Delta m_1}{\rho \Delta t} = \frac{\Delta m_2}{\rho \Delta t} = \dots \dots \dots = \frac{\Delta m_n}{\rho \Delta t}$$

NOTE

ρ and Δt
cancel out for
all terms.
 $\rho = \text{constant}$
For ideal fluid

$$\Rightarrow \Delta m_1 = \Delta m_2 = \Delta m_3 = \dots = \Delta m_n$$

This proves that equation of continuity is the law of conservation of mass.

Q. 4: Derive mathematical equation for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?

Ans. Bernoulli's Equation:

This equation states that for an ideal fluid in a steady flow, the sum of pressure, gravitational potential energy and kinetic energy remains constant.

OR

The total energy of a liquid flowing from one point to another point remains constant.

OR

The work done on a liquid is equal to the change in energy of that liquid.

$$W = \Delta E$$

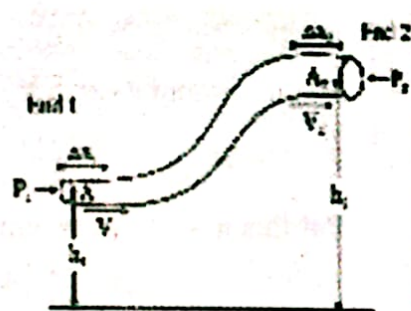
Etymology (اسم) or History:

This equation is named after Daniel Bernoulli who published it in his book Hydrodynamics in 1738.

Derivation of Mathematical Equation:

Consider an ideal fluid in a steady flow through a pipe of non-uniform cross-sectional area as shown in figure.

The lower end of pipe is at height h_1 while the upper end is at height h_2 .



A fluid is moving along through a non-uniform pipe. The velocity of the fluid section on the left is equal to the velocity of the fluid section on the right.

According to the law of conservation of energy

$$W = \Delta E$$

OR

$$W = \Delta K.E + \Delta P.E \text{ ----- (i)}$$

$$\text{Where } W = W_1 + W_2 \text{ ----- (ii)}$$

W_1 = work done on lower end of pipe

W_2 = Work done on upper end of pipe

$$W_1 = \overline{F_1} \Delta x_1 \quad \text{By definition of work}$$

$$W_1 = F_1 \Delta x_1 \cos \theta, \text{ But } \theta = 0^\circ, \cos 0^\circ = 1$$

$$\Rightarrow W_1 = F_1 \Delta x_1 \text{ ----- (iii)}$$

$$W_2 = \overline{F_2} \Delta x_2$$

OR

$$W_2 = F_2 \Delta x_2 \cos \theta, \theta = 180^\circ, \cos 180^\circ = -1$$

$$W_2 = -F_2 \Delta x_2 \text{ ----- (iv)}$$

Put equation (iii) and (iv) in equation (ii)

$$W = W_1 + W_2$$

$$\Rightarrow W = F_1 \Delta x_1 - F_2 \Delta x_2$$

NOTE

$$P = \frac{F}{A}$$

$$\text{Or } F = PA$$

$$\Rightarrow W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

$$\Rightarrow W = P_1 (A_1 \Delta x_1) - P_2 (A_2 \Delta x_2)$$

$$\Rightarrow W = P_1 \Delta V_1 - P_2 \Delta V_2$$

$$\Rightarrow W = (P_1 - P_2) \Delta V \text{ ----- (v)}$$

But

$$\rho = \frac{\Delta m}{\Delta V}$$

OR

$$\Delta V = \frac{\Delta m}{\rho} \text{ Put in equation (v)}$$

$$\begin{aligned} A_1 \Delta x_1 &= \Delta V_1 \\ A_2 \Delta x_2 &= \Delta V_2 \text{ From continuity} \\ \text{but } \Delta V_1 &= \Delta V_2 \\ \text{equation} \end{aligned}$$

$$W = (P_1 - P_2) \frac{\Delta m}{\rho} \text{ ----- (vi)}$$

Also

$$\Delta K.E = K.E_f - K.E_i$$

OR

$$\Delta K.E = \frac{1}{2} \Delta m_2 v_2^2 - \frac{1}{2} \Delta m_1 v_1^2 \text{ ----- (vii)}$$

Also

$$\Delta P.E = \Delta m_2 g h_2 - \Delta m_1 g h_1 \text{ ----- (viii)}$$

Put equation (vi), (vii) and (viii) in equation (i)

$$(P_1 - P_2) \frac{\Delta m}{\rho} = \frac{1}{2} \Delta m_2 v_2^2 - \frac{1}{2} \Delta m_1 v_1^2 + \Delta m_2 g h_2 - \Delta m_1 g h_1$$

 From continuity equation $\Delta m_2 = \Delta m_1 = \Delta m$

 Therefore all Δm 's cancel out

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$$

OR

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

Which is the required Bernoulli's Equation.

Conservation of Energy and Bernoulli's Equation:

Bernoulli's equation is another form of the law of conservation of energy.

According to Bernoulli's equation

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

 Divide both sides by ρ

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 + g h_1 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2 + g h_2$$

As

$$\rho = \frac{m}{V} \text{ put in this equation}$$

$$\frac{P_1}{\frac{m}{V_1}} + \frac{1}{2} v_1^2 + g h_1 = \frac{P_2}{\frac{m}{V_2}} + \frac{1}{2} v_2^2 + g h_2$$

$$\frac{P_1 \cdot V_1}{m} + \frac{1}{2} v_1^2 + g h_1 = \frac{P_2 \cdot V_2}{m} + \frac{1}{2} v_2^2 + g h_2$$

$$\text{OR } \frac{F_1}{A_1} \cdot \frac{V_1}{m} + \frac{1}{2} v_1^2 + gh_1 = \frac{F_2}{A_2} \cdot \frac{V_2}{m} + \frac{1}{2} v_2^2 + gh_2$$

$$\text{But } V = A \Delta x$$

$$\text{So } \frac{F_1 \cdot \cancel{A_1} \Delta x_1}{\cancel{A_1} m} + \frac{1}{2} v_1^2 + gh_1 = \frac{F_2 \cdot \cancel{A_2} \Delta x_2}{\cancel{A_2} m} + \frac{1}{2} v_2^2 + gh_2$$

Multiply both sides by m

$$F_1 \Delta x_1 + \frac{1}{2} m v_1^2 + mgh_1 = F_2 \Delta x_2 + \frac{1}{2} m v_2^2 + mgh_2$$

One can check that the dimensions of the terms

$$F \cdot \Delta x + \frac{1}{2} m v^2 + mgh_1 = [ML^2T^{-2}]$$

Which are the dimensions of energy which remains conserved.

Therefore one can say that Bernoulli's equation is the law of conservation of energy.

Q. 5: Using Bernoulli's equation, what is the speed of efflux from a leak at the bottom of a large storage tank?

Ans. Speed of Efflux:

Consider a large storage tank which has a leak at the bottom as shown in figure. The atmospheric pressure is $P_1 = P_2 = P$ same while $h = h_1 - h_2$, $V_1 = 0$ because it is very very small compared to V_2 .

Applying Bernoulli's equation

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$$

$$P_1 - P_2 = 0, \quad V_1 = 0$$

$$\Rightarrow \rho gh_1 = \frac{1}{2} v_2^2 + \rho gh_2$$

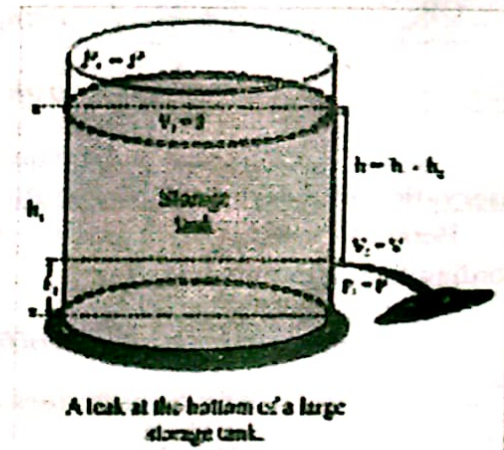
$$\Rightarrow \rho gh_1 - \rho gh_2 = \frac{1}{2} v_2^2$$

$$\text{OR } \rho g(h_1 - h_2) = \frac{1}{2} v_2^2$$

$$\text{OR } \rho g(h) = \frac{1}{2} v_2^2$$

$$\text{OR } v_2^2 = 2gh$$

$$\text{OR } v_2 = \sqrt{2gh}$$



This is the speed of efflux from a leak at the bottom of a large storage tank. The speed v_2 is exactly equally to the speed of a freely falling body. This is called Torricelli's Theorem.

6: By Bernoulli's equation how we can determine the speed of the fluid in a pipe?

Ans. Determining Speed of Fluid in a Pipe:

The device which is used to determine the speed of fluid in a pipe is called venturi meter.

Principles of Venturi Meter:

It works on Bernoulli's principles to measure the speed or rate of flow of a fluid through a pipe.

Construction of Venturi Meter:

It is a pipe of narrower and wider sections or of smaller and larger cross-sectional area. The narrow part of the pipe is called throat.

The velocity of fluid at end_1 is $\bar{V}_1$ and at the end_2 it is $\bar{V}_2$. The pipe is horizontal therefore its potential energy is taken zero.

$$\text{Or } \rho gh_1 = \rho gh_2 = \rho gh$$

$$\text{Or } \rho gh_1 - \rho gh_2 = 0$$

Applying Bernoulli's equation

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho gh_2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2$$

$$P_1 - P_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) \text{----- (i)}$$

From continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\text{Or } V_2 = \frac{A_1 V_1}{A_2} \text{----- (ii)}$$

Put this in equation (i)

$$P_1 - P_2 = \frac{1}{2} \rho \left[\left(\frac{A_1 V_1}{A_2} \right)^2 - V_1^2 \right]$$

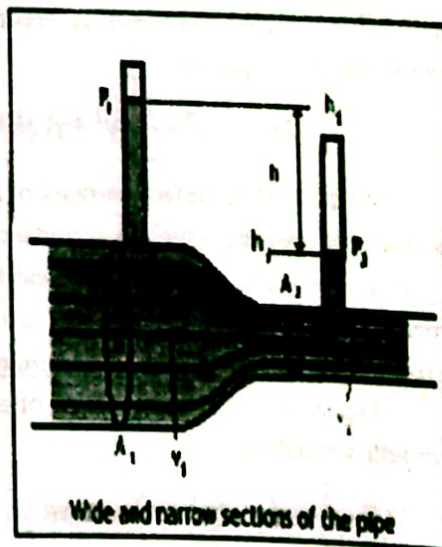
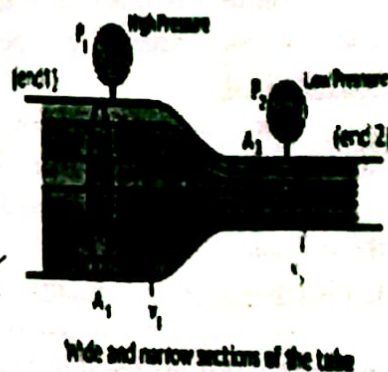
$$P_1 - P_2 = \frac{1}{2} \rho \left[\frac{A_1^2}{A_2^2} - 1 \right] V_1^2$$

$$\text{Or } P_1 - P_2 = \frac{1}{2} \rho \left[\frac{A_1^2 - A_2^2}{A_2^2} \right] V_1^2$$

$$\text{Or } V_1^2 = \frac{2(P_1 - P_2)}{\rho \left[\frac{A_1^2 - A_2^2}{A_2^2} \right]}$$

$$\text{Or } V_1 = \sqrt{A_2^2 \cdot \frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}}$$

$$\text{Or } V_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}}$$



P_1 and P_2 can be measured by barometer while ρ , A_1 and A_2 are easy to calculate.

Also $P_1 - P_2 = \Delta P = \rho gh$

$$v_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$v_1 = A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

Q. 7: What is aero-foil? Explain aero-foil lift on the wing of an aeroplane.

Ans. Aero-Foil:

The devices which are shaped or designed so that the relative motion between it and the fluid produces a force perpendicular to the flow are called aero-foils.

Aero-Foil Lift:

The component of force which acts on the aero-foil perpendicular to the direction of motion is called aero-foil lift.

Aero-foil lift is produced due to the pressure difference of the two surfaces of the aero-foil.

Explanation of Aero-Foil Lift:

The shape of aero-foil is designed in such a way that the speed of fluid at the top surface is higher than the speed of fluid at the bottom surface. This is because, the particles of fluid come closer at the top surface decreasing its effective area which in accordance with continuity equation increases the speed of fluid. Due to the increased speed at the top surface its pressure decreases in accordance with Bernoulli's equation:

$$\text{i.e. } P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$$

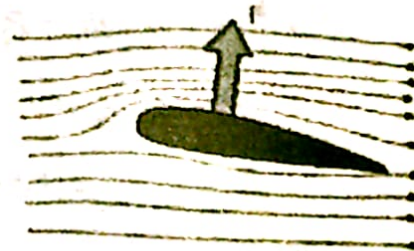
At the bottom surface particles of fluid are far-away and nearly equally spaced has a higher pressure which in accordance with Bernoulli's equation, this difference in pressure between the top surface and bottom surface gives rise to an upward lift, which is called aero-foil lift. This acts in a direction perpendicular to fluid flow.

Aero-Foil Lift on the Wings of Aeroplane:

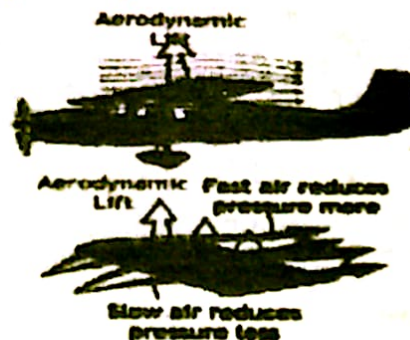
The aero-foil lift on the wings of an aeroplane is produced in accordance with Bernoulli's equation:

$$\text{i.e. } P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$$

The wings of an aeroplane are so shaped that the speed of fluid at the top surface is higher than the speed of fluid at the bottom surface. This is because the



Stream lines are crowded together above the aero-foil, so flow speed is higher and pressure is low. Because of this decreased pressure a lift is exerted



streams of fluid come close at the top surface decreasing its effective area which in accordance with continuity equation increases the speed of fluid (air). Due to the increased speed at the surface its pressure decreases in accordance with Bernoulli's equation.

At the bottom of the wing of the aeroplane due to its shape the streams of fluid are equally spaced has a higher pressure in accordance with Bernoulli's equation.

This difference in pressure at the top and bottom surface of the wings of aeroplane gives rise to an upward lift and the wing is lifted upward.

Q. 8: Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray.

Ans. According to Bernoulli's principle (equation), the sum of pressure, K.E per unit volume and potential energy per unit volume is constant:

$$\text{i.e. } P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$

Increasing one of the variable, decreases the other to satisfy the equality.

Engine Carboretor:

It is a device which mix air and fuel for the internal combustion in an engine.

Working Principles:

It works on Bernoulli's principles which states that, the sum of pressure K.E per unit volume, P.E per unit volume for an ideal fluid in a steady flow is constant.

$$\text{i.e. } P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$

Construction:

It is made of a tube with a constriction (throat) in it.

Working:

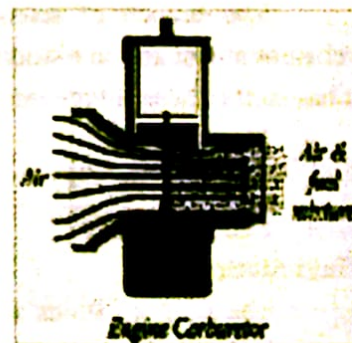
When air is drawn into the thicker part of the tube it has same pressure as the fuel in the fuel supply (tank). When the air passes through the constriction (smaller cross-sectional area) its speed increases due to continuity equation. Due to the increased speed its pressure decreases in accordance to Bernoulli's equation.

The drop in pressure compared to higher pressure on the fuel in fuel supply. The fuel is drawn into the tube and is mixed with air and is used for combustion in the engine.

In newer cars electronic fuel injectors (EFI) rather than carburetors are used.

Perfume Spray Bottle:

It is an atomizing device which emits perfume liquid in the form of fine spray.



Where η is constant of proportionality and is called co-efficient of viscosity of fluid, $\frac{v}{x}$ in this equation is called shear rate. This equation can also be expressed

as: $\eta = \frac{Fx}{Av} \quad \left(\text{As } v = \frac{x}{t} \right)$

$$\eta = \frac{Fx}{A \left(\frac{x}{t} \right)}$$

Or $\eta = \frac{Ft}{A}$

Nature: It is a scalar quantity.

Unit of η :

The SI unit of co-efficient of viscosity according to equation $\eta = \frac{Ft}{A}$ is Pascal second or Pa s.

The CGS unit of η is called poise named after French Physiologist Jean Louis Poiseuille.

$$\text{One poise} = \frac{1 \text{ dyne}}{\text{cm}^2}$$

Or $\text{One poise} = 0.1 \text{ PaS}$

Dimensions:

The dimensions of co-efficient of viscosity according to equation $\eta = \frac{Ft}{A}$ are $[ML^{-1}T^{-1}]$.

Factors upon which η Depends:

- Temperature of fluid
- Nature of fluid

Q. 3: Use Bernoulli's Equation to explain the working of filter pump and the blood flow.

Ans. Filter Pump:

It is a device which produce partial vacuum in a vessel attached to it.

Principles:

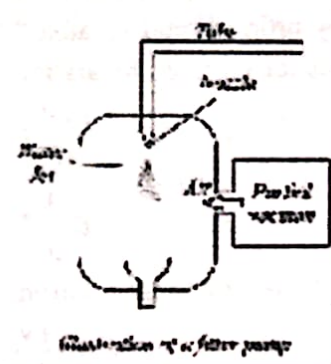
It works on the Bernoulli's equation which states that for an ideal fluid in a steady flow the sum of pressure. K.E per unit volume, P.E per unit volume is constant.

$$P + \frac{1}{2}v^2 + \rho gh = \text{Constant}$$

Construction:

It consists of a tube with jet connected to it in which water flows from the tube toward the jet.

Working:



When water passes from the jet section its speed increases in accordance with Bernoulli's equation, because pressure decreases near it. This drop in pressure allows air to flow in from the side tube connected to the vessel. In this way air and water are forced together at the bottom of filter pump and partial vacuum is created in the vessel attached.

Blood Flow:

Blood is that liquid which circulate in the arteries and veins of humans and other vertebrates for the exchange of oxygen and carbon dioxide. It is a viscous liquid having co-efficient of viscosity as $1.6 \times 10^{-3} \text{ Pa s}$.

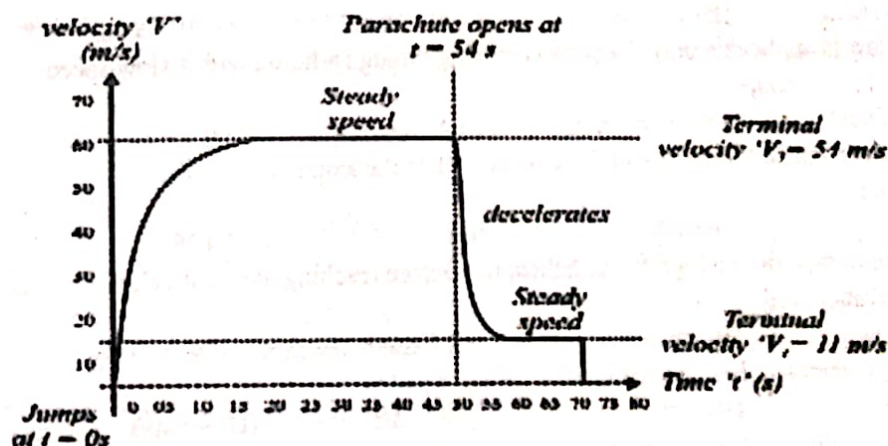
Role of Continuity Equation in Blood Flow:

The speed of blood depends upon the radius of arteries and veins. According to continuity equation smaller the radius of the arteries higher is the blood speed, and vice versa. Arteriosclerosis is that disease which make the artery walls thick and hard which reduce volume flow rate of blood. The building up of cholesterol also plays that role.

Role of Bernoulli's Equation in Blood Flow:

Due to arteriosclerosis and higher level of cholesterol increases blood speed and decreases pressure blood pressure in accordance with Bernoulli's equation. For normal working of the human body the pressure must be maintained to cause the same flow rate. The heart has to work hard to maintain the same flow rate. The drop in pressure can dislodge the plague which can then block smaller arteries. Which become a cause for heart attack.

Q.4: Do You Know? In a free fall the paratrooper attains his terminal velocity twice, once before opening his parachute and the other after opening the chute. Without opening the chute, paratrooper offers lower radius to air and therefore has a high terminal speed. Whereas after opening the chute he has large radius thereby having sufficiently low terminal speed to allow him to fall safely on the ground. For example, consider the graph below which explain the motion of paratrooper.



Discussion:

The motion of paratrooper is discussed for different stages as follow;

- Stage 1: At $t = 0$ s After just he jumps from the plane, the skydiver does not move very fast and his weight is a bigger force than air resistance, so he accelerates downwards.
- Stage 2: At $t = 19$ s The force of air resistance has now increased so much that it is equal to the skydiver's weight. So the forces are balanced and the speed becomes constant (this is terminal velocity 1)
- Stage 3: At $t = 54$ s When he opens the chute air resistance increases considerably; the air resistance force is much greater than the weight force, so the skydiver slows down.
- Stage 4 at $t = 48$ s The skydiver slows down now, the air resistance is reduced, until it becomes equal to the weight. The forces are balanced and the speed remains constant (this is terminal velocity 2) .

Q.5 What would happen to a plane sheet of paper if air is blown over it?

Ans Cut a long piece of paper, blowing over top of paper, make the paper rise. As air is blown over the top of paper its speed is greater than the speed at the bottom. In the high speed region (that is at top) the pressure is reduced due to Bernoulli's principles to fill that reduced pressure region the air from the bottom rushes to maintain the constant atmospheric pressure thus lifting the paper with itself.



MCQ's

1. For substances that do not flow easily (like honey) have a value for the coefficient of viscosity.
(A).Low (B).High (C).Zero (D).Negative
2. A unit for viscosity, the centipoise, is equal to which of the following?
(A). 10^{-3} Ns/m² (B). 10^{-2} Ns/m² (C). 10^2 Ns/m² (D). 10^3 Ns/m²
3. Stok's law is applicable only if a body is moving through a liquid with a slow speed and has shape.
(A).A cubical (B).A spherical (C).A rough (D).Any
4. The net force that acts on a 10-N falling object, when it encounters 4 n of air resistance is
(A).0 N (B).4 N (C).6 N (D).10 N
5. A skydiver jumps from a high-flying helicopter. Before reaching terminal velocity, her acceleration will
(A).Increase (B).Decrease (C).Remain the same (D).Be zero
6. At terminal velocity, the acceleration of a falling object is
(A).0 m/s² (B).1 m/s² (C).-9.8 m/s² (D).9.8 m/s²
7. According to the equation of continuity $Av = \text{constant}$. This constant is equal to
(A).Volume of fluid (B).Mass of fluid (C).Density of fluid (D).Volume flow rate

8. At the constriction in the cross-section for ideal flow, from equation of continuity it follows that the speed of fluid is
(A).Greater (B).Smaller (C).Same (D).Zero
9. As water in level pipe passes from a narrow cross-section of pipe to a wider cross section, the pressure against the wall
(A).Increase (B).Decrease (C).Remain the same (D).Is zero
10. A 4 m high tank filled with water is drilled with four identical small holes at 1 m, 1.5 m, 2 m and 2.5 m from the bottom of tank, the speed of efflux will be greater from the hole at.....
(A).1 m (B).1.5 m (C).2 m (D).2.5 m
11. Venturi meter is a device used to measure the.....
(A).Mass of fluid (B).Viscosity of fluid (C).Speed of fluid (D).Density of fluid
12. A certain pipe has a cross-section area of 0.0001 m^2 in which water is flowing at 10 m/s. The flow rate is
(A). $0.00001 \text{ m}^3/\text{s}$ (B). $0.001 \text{ m}^3/\text{s}$ (C). $1 \text{ m}^3/\text{s}$ (D). $10.0001 \text{ m}^3/\text{s}$
13. At sufficiently high speeds the flow of viscous fluid becomes
(A).Unexpected (B).Streamline (C).Non-viscous (D).Turbulent
14. The water in the tank is 10 m above the leak point. The speed with which the water emerges from the leak is
(A).10 m/s (B).14 m/s (C).194 m/s (D).0.1 m/s
15. When the radius of the artery is reduced, the blood pressure...
(A).Increases (B).Decreases (C).Remains constant (D).Is zero

Answer Keys:

1	B	5	B	9	A	13	D
2	A	6	A	10	A	14	B
3	B	7	D	11	C	15	B
4	C	8	A	12	B		

MCQs SolutionQ1. Ans. **B. High**Q2. **Solution:** We know that

$$1 \text{ centipoise} = 10^{-3} \text{ Pascal} \cdot \text{sec}$$

$$1 \text{ centipoise} = 10^{-3} \frac{\text{N}}{\text{m}^2} \text{ sec}$$

$$1 \text{ centipoise} = 10^{-3} \text{ N sec} / \text{m}^2$$

$$\text{Ans. } \boxed{A. 10^{-3} \text{ N s} / \text{m}^2}$$

Q3. From Stoke's law

$$F_1 = 6\pi\eta rv$$

Here r is the radius of spherical body

So

Ans. **B. Spherical**Q4. **Solution:**

A body of weight falling down in a resistance medium (air), weight "w" is acting downward and the air resistance is acting upward, so the net force is given by:

$$F_{net} = W - F_{air} = 10 - 4 = 6N$$

Ans. C. 6N

Q5. When an object is falling downward the net force on it is given by:

$$F_{net} = W - F_D$$

$$ma_{net} = mg - ma_D$$

$$a_{net} = g - a_D$$

$$a_{net} = 9.8 - a_D \text{ ----- (i)}$$

Ans. B. Decrease

When body is falling downward the drag acceleration (a_D) increases as a result the net acceleration will decrease.

Q6. Solution:

When an object attains terminal velocity at that point the total force will be zero on the object. Then according to Newton's second law if the force is zero, acceleration will always be zero.

Ans. A. $0m/sec^2$

Q7. Solution:

According to equation of continuity volume flow rate is equal to "AV"

$$\frac{\Delta V}{\Delta t} = Av$$

$$\frac{\Delta V}{\Delta t} = Av = \text{constant}$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \text{constant}$$

Ans. D. Volume flow rate

Q8. Solution:

From equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

If $A_1 v_1 = \text{constant}$

$$\Rightarrow v_2 \propto \frac{1}{A_2} \text{--- (i)}$$

From relation (i) " v_2 " is inversely proportional to " A_2 " if " A_2 " decreases " v_2 " will increase so:

Ans. **A. Greater**

Q9. **Solution:**

According to equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

As " A_2 " increases therefore " v_2 " will decrease.

According to Bernoulli's principle:

$$\text{Pressure} \propto \frac{1}{\text{Velocity}}$$

If velocity decreases pressure will increase in that region.

Ans. **A. Increases**

Q10. **Solution:**

As

$$\text{height} = h = 4m$$

$$h_1 = 1m \quad \text{From bottom}$$

$$h_2 = 1.5 \quad \text{From bottom}$$

$$h_3 = 2.5 \quad \text{From bottom}$$

We know that

$$v = \sqrt{2gh_{net}}$$

First Hole

$$v = \sqrt{2g(h - h_1)}$$

$$v = \sqrt{2g(4 - 1)}$$

$$v = \sqrt{2g(3)}$$

$$v_1 = \sqrt{6g}$$

$$v_1 = \sqrt{6(9.8)}$$

$$v_1 = 7.7 \text{ m/sec}$$

2nd Hole

$$v = \sqrt{2g(h - h_2)}$$

$$v = \sqrt{2g(4 - 1.5)}$$

$$v = \sqrt{2g(2.5)}$$

$$v_2 = \sqrt{5g}$$

$$v_2 = \sqrt{5(9.8)}$$

$$v_2 = 7 \text{ m/sec}$$

4th Hole

$$v = \sqrt{2(g)(h - h_4)}$$

$$v_4 = \sqrt{2(9.8)(4 - 2.5)}$$

$$v_4 = \sqrt{2(9.8)(1.5)}$$

$$v_4 = 5.4 \text{ m/sec}$$

Therefore speed is high for 1st hole $h_1 = 1\text{m}$

Ans. A

Q11.

Ans. C. Speed of fluid

Q12.

Solution:

$$A = 0.001 \text{ m}^2$$

$$v = 10 \text{ m/sec}$$

$$\frac{\Delta V}{\Delta t} = ?$$

As we know that

$$\frac{\Delta V}{\Delta t} = Av$$

Putting v

$$\frac{\Delta V}{\Delta t} = (0.0001)(10)$$

3rd Hole

$$v = \sqrt{2g(h - h_3)}$$

$$v = \sqrt{2g(4 - 2)}$$

$$v = \sqrt{2g(2)}$$

$$v_3 = \sqrt{4g}$$

$$v_3 = \sqrt{4(9.8)}$$

$$v_3 = 6.3 \text{ m/sec}$$

A. 1m

values

$$\frac{\Delta V}{\Delta t} = 0.001 \text{ m}^3/\text{sec}$$

Ans. **B. 0.001 m³/sec**Q13. Ans. **D. Turbulent**

$$h = 10 \text{ m}$$

Q14.

$$v = ?$$

We know that

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(9.8)(10)}$$

$$v = 14 \text{ m/sec}$$

Ans. **B. 14 m/sec**

Q15. From equation of continuity

$$Av = \text{Constant}$$

$$\pi r^2 v = \text{Constant}$$

$$v = \frac{\text{Constant}}{\pi} \frac{1}{r^2}$$

$$v \propto \frac{1}{r^2}$$

When radius of artery decreases velocity of blood increases.

Now by Bernoulli's principle if velocity increases pressure will decrease.

Ans. **B. Decreased**Short Questions Answers

Q. 1: From the top of a tall building, you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?

Ans. Thesis Statement:

The table-tennis ball filled with air reaches terminal velocity first because of its lower mass as compared to the water-filled ball.

Explanation:

The terminal velocity of a spherical body is given by:

$$v_r = \frac{2\rho g r^2}{9\eta}$$

Since $\frac{2gr^2}{9\eta} = \text{constant } r^2$

$$\Rightarrow v_r \propto \rho \rightarrow (i)$$

This shows that terminal velocity is directly proportional to the density of the spherical body.

Furthermore,

$$\rho = \frac{m}{V} \quad \text{Putting in equation (i)}$$

$$v_T \propto \frac{m}{V} \quad (\text{Volume of balls} = \text{constant})$$

$$\Rightarrow v_T \propto m$$

Thus it is clear that greater the mass of a body higher will be its terminal velocity and longer will it take to reach terminal velocity.

Conclusion:

Thus from the above discussion it can be concluded that the ball filled with air has less mass and therefore reaches terminal velocity first.

Q. 2: Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Ans. Thesis Statement:

A squirrel can jump from a tree branch and runaway because of its larger surface area exposed to air due to which the drag force on the squirrel is high and it lands on the ground with low velocity.

Secondly, the smaller mass of the squirrel and its elastic body saves it from damage.

Explanation:

When a squirrel jumps from a tree, it spreads its body and the surface area exposed to air is large as compared to the size and weight of the squirrel is high due to which its velocity decreases and a smooth and safe landing is possible.



Whereas, when a man jumps from a tree, his surface area exposed to air is relatively small as compared to his weight. Therefore the drag force on him is small and his velocity is greater which can break his bone.

Conclusion:

A squirrel can jump from a tree branch to the ground and runaway undamaged due to large surface area less mass while a man can break a bone due to small surface area and greater weight.

Q. 3: How does the terminal speed of a parachutist before opening a parachute. Compare to the terminal speed afterward? Why is there a difference?

Ans. Thesis Statement:

The terminal speed of a parachutist before opening the parachute is much higher than the terminal speed after opening the parachute.

Reason:

The terminal speed of the parachutist with closed parachute is high because of his less surface area exposed to air as compared to the terminal speed of the

parachutist after opening the parachute because of increased surface area exposed to air.

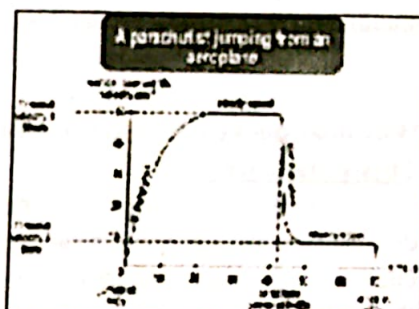
Explanation:

The terminal velocity is given by:

$$v_T = \frac{F_g}{6\pi\eta r}$$

$$v_T \propto \frac{1}{r} \rightarrow (i)$$

Where (r) is the radius of the falling body



v_T Before Opening the Parachute:

Since the terminal velocity is directly proportional to the radius of the body.

i.e. $v_T \propto \frac{1}{r}$

Before opening the parachute radius (r) of the parachutist is very small as a result his (v_T) is very high.

v_T After Opening the Parachute:

Since $v_T \propto \frac{1}{r}$ when the parachutist open the parachute the radius of the area exposed to air is increased. As a result his (v_T) terminal velocity is small.

Q. 4: You can squirt water over a greater distance by placing your thumb over the end of a garden hose, than by leaving it completely uncovered. Explain how this works?

Ans. Thesis Statement:

One can squirt water over a greater distance by placing one's thumb over the end of a garden hose, because the area of cross-section is decreased as a result the speed of water increases.



Explanation:

According to the equation of continuity

$$AV = \text{constant}$$

$$\Rightarrow V = \frac{\text{Constant}}{A}$$

$$\Rightarrow V \propto \frac{1}{A}$$

A = area of cross-section
V = Volume of fluid flowing

This equation clearly shows that the speed of flow for a fluid "V" is inversely proportional to the area of cross-section.

Figurative Interpretation:

- In case-1 the area through which the water is flowing is large and water moves slowly with " V_1 " and covers a small distance " d_1 ".
- In case-2 when finger is placed on the end of the hose (pipe), the area decreases and as result the velocity " V_2 " of the water increases and covers a greater distance " d_2 ".

Conclusion:

Thus, one can squirt (پھینکا) water over a greater distance by placing one's thumb over the end of a garden hose because of its decreased cross-sectional area.

Q. 5: Why does smoke rise faster in a chimney on a windy day?

Ans. Thesis Statement:

Smoke rises faster in a chimney on a windy day because the air pressure at the upper end of the chimney is low as compared to the lower end due to the high speed of air at the upper end and the smoke moves upwards.

**Explanation:**

On a windy day air is blowing with high speed, at the upper end of chimney due which pressure is reduced.

Whereas, at the lower end, the air pressure is high because of low air speed i.e.

$$P_{upper} < P_{lower}$$

- Due to this pressure difference smoke moves in the chimney on a windy day.
- In principles air moves from place of high pressure to a place of low pressure.

Conclusion:

Thus it is concluded that smoke rise faster in a chimney on a windy day due to the pressure difference between lower and upper end of the chimney.

Q. 6: Two boats moving in parallel paths close to one another risk colliding. Why?

Ans. Thesis Statement:

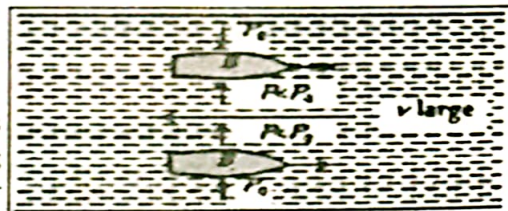
Two boats moving in parallel paths close to one another risk colliding due to lower water pressure of the stream of water between them.

Explanation:

From Bernoulli's principle

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$

It is clear that increasing speed "V" means decreasing pressure "P". When the boats are moving in parallel paths close to one another the water touching the boats is also moving with a high speed.



The speed of water between the boats is higher than the speed of water on the outer sides of the boats. Due to this speed the pressure decreases and the boats are pulled towards each other and may collide.

Conclusion:

The conclusion is that two boats moving in parallel paths close to one another risk colliding due to reduced water pressure in the water stream between them.

Q. 7: A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?

Ans. Thesis Statement:

A cricket ball moves past an observer from left to right, spinning counter clockwise will tend to deflect away from the observer.

Explanation:

When a ball moves from left to right an observer and spinning counter (anti) clockwise. The air is moving opposite to the direction of the ball.

Now, the side of the ball towards (کی طرف) the observer is spinning in opposite direction of the air. Therefore the air speed decreases while, on other side of the ball (away from observer) the spin of the ball and air are in the same direction due to which speed of the air increases.

Applying Bernoulli's Principle:

According to this principle

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$

This means that increasing speed of air (fluid) at a point will decrease air pressure at that point.

Therefore the pressure of air on the side of the ball toward the observer is higher than the pressure on the side away from the observer. Due to this pressure difference the ball will tend to move away from the observer.

Conclusion:

Thus the conclusion is that if a cricket ball moves past an observer from left to right, spinning counter clockwise will tend to deflect away from the observer.

Q. 8: If aero-foil lift the aero-plane in upright position, how do the pilots make the aero-planes fly upside down?

Ans. Thesis Statement:

Pilot make the aeroplane fly upside down by increasing the angle of attack by pointing the nose of the plane up.

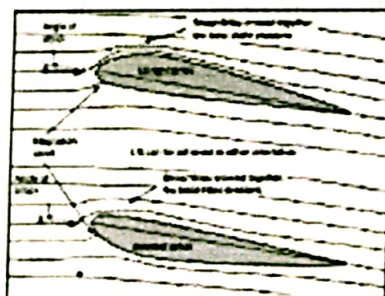
Explanation:

When an aeroplane flies the angle of attack of the wings provides the lift (up and down) the wings of fighter planes and aerobatics can flip (مڑنا) upwards as well as downwards.

When a plane is flying upside down the effect of the aero-foil lift is reversed (i.e. downward). But the pilot can counter this downward aero-foil lift by adjusting the wings in opposite direction and by pointing the nose of the plane towards the sky.

Conclusion:

Thus it is concluded that pilots can make the aeroplane fly upside down by keeping the angle of attack skywards and by flipping (turning) the wings in opposite direction.



Q. 9: Why do the golf balls have dimples?

Ans. Thesis Statement:

Golf balls have dimples due to which the drag on the ball is reduced and the ball can travel a longer distance.

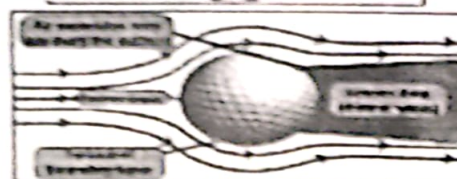
Explanation:

For a smooth surface ball the wake behind (پچھل طرف) is large due to which the low pressure region is greater than the high pressure region and the ball gets a strong backward pull.

For a dimpled ball, the streamline flow remains connected to the surface of the ball for longer time due to which the wake behind the ball (low pressure region) becomes small and backward pull on the ball is very weak i.e. drag is reduced.

Conclusion:

The golf balls have dimples in order to reduce drag force and thus the ball travels a longer distance.



Q. 10: How by using wind deflectors on the top truck cabs reduce fuel consumption?

Ans. Thesis Statement:

Wind deflectors on the top of truck cabs reduce fuel consumption by streamlining the flow of air around the vehicle.

Explanation:

When truck are moving with high speed the drag force on them is large in order to overcome this drag fuel consumption is high the drag on the truck can be reduced by shaping it in such a way to streamline the air flow on its sides. The smooth (streamline) surface help the air to slip fast along it and thereby reduces the drag on the vehicle. Therefore the fuel consumption is reduced.



Conclusion:

Wind deflector on the top of truck's cab stream lines the air flow around the truck which decreases the drag force which in turn reduces fuel consumption.

Assignments

Assignment 6.1: A certain globular protein particle has a density of 1246 kg m^{-3} . It falls through water (having coefficient of viscosity $8.91 \times 10^{-4} \text{ Pa.s}$) with a terminal speed of $8.33 \times 10^{-4} \text{ m.s}^{-1}$. Find the radius of the particle.

Given Data:

Density of protein particle = $\rho = 1246 \text{ kg m}^{-3}$

Co-efficient of viscosity of water = $\eta = 8.91 \times 10^{-4} \text{ Pa.s}$

Terminal speed of protein particle $= v_T = 8.33 \times 10^{-6} \text{ m/sec}$

Required Data:

Radius of the particle $= r = ?$

Solution:

We know that

$$v_T = \frac{2g\rho r^2}{9\eta}$$

$$v_T 9\eta = 2g\rho r^2$$

$$r^2 = \frac{9\eta v_T}{2g\rho} \text{ ----- (i)}$$

Putting values in equation (i)

$$r^2 = \frac{9(8.91 \times 10^{-4})(8.33 \times 10^{-6})}{2(9.8)(1246)}$$

$$\sqrt{r^2} = \sqrt{0.0273 \times 10^{-10}}$$

$$r = 0.165 \times 10^{-5} \text{ m}$$

$$r = 1.65 \times 10^{-6} \text{ m}$$

Result:

Radius of particle $= r = 1.65 \times 10^{-6} \text{ m}$
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Assignment 6.2: The heart pumps blood into the aorta, which has an inner radius of 1.0 cm. The aorta feeds 32 major arteries (each have an inner radius of 0.21 cm). If blood in the aorta travels at a speed of 25 cm/s, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid.

Given Data:

Radius of aorta $= r_1 = 1.0 \text{ cm} = 0.01 \text{ m}$

Radius of artery $= r_2 = 0.21 \text{ cm} = 0.0021 \text{ m}$

Blood speed in aorta $v_1 = 25 \text{ cm/sec} = 0.25 \text{ m/sec}$

Area of cross-section of aorta $= A_1 = \pi r_1^2$

Area of cross-section of an artery $= A_2 = \pi r_2^2$

Area of cross-section of 32 arteries $= A_2 = 32\pi r_2^2$

Required Data:

Blood speed in artery $= v_2 = ?$

Solution:

From equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\cancel{\pi} r_1^2 v_1 = 32 \cancel{\pi} r_2^2 v_2$$

$$r_1^2 v_1 = (32) r_2^2 v_2$$

$$v_2 = \frac{r_1^2 v_1}{(32) r_2^2}$$

Putting values

$$v_2 = \frac{(0.01)^2 (0.25)}{(32)(0.0021)^2}$$

$$v_2 = 0.18 \text{ m/sec}$$

Result:

$$\text{Blood speed in artery} = v_2 = 0.18 \text{ m/sec}$$

Assignment 6.3: Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1} , while at another point 3 m higher, the speed is 4 ms^{-1} . At lower point the pressure is 80 kPa. Find the pressure at the upper point.

Given Data:Speed of water at one point = $V_1 = 3 \text{ m/sec}$ Speed of water at another point = $V_2 = 4 \text{ m/sec}$ Initial height = $h_1 = 3 \text{ m}$ Height at other end = $h_2 = 3 \text{ m}$ Pressure at lower point = $P_1 = 80 \text{ kPa} = 80 \times 10^3 \text{ Pa}$ Density of water = $\rho = 1000 \text{ kg/m}^3$ Required Data:Pressure of upper point = $P_2 = ?$ Solution:

From Bernoulli's equation

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 - \frac{1}{2} \rho V_2^2 - \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2} \rho (V_1^2 - V_2^2) + \rho g (h_1 - h_2)$$

Putting values

$$P_2 = (80 \times 10^3) + \frac{1}{2} (1000) (3^2 - 4^2) + (1000) (9.8) (0 - 3)$$

$$P_2 = 80 \times 1000 + \frac{1000}{2} (9 - 16) + (1000) (9.8) (-3)$$

$$P_2 = 1000 \left[80 - \frac{7}{2} - 29.4 \right] = 1000 (47.1)$$

Result:

$$P_2 = 147.1 \times 10^3 \text{ Pa} = 147.1 \text{ KPa}$$

Assignment 6.4: A tank full of water has a (small) hole near its bottom at a depth of 2.0 m from the top surface, which is open to air. What is the speed of the stream of water emerging from the hole?

Given Data:Depth of hole = $h = 2 \text{ m}$ Required Data:Speed of efflux of water = $V = ?$

Solution:

As we know that

$$v = \sqrt{2gh} \quad \text{Putting values}$$

$$v = \sqrt{2(9.8)(2)}$$

$$v = \sqrt{39.2}$$

$$v = 6.3 \text{ m/sec}$$

Result:

$$\text{Speed of efflux of water} = v = 6.3 \text{ m/sec}$$

Assignment 6.5: A venturi meter is measuring the flow of water in a pipe having cross-sectional area of 0.0038 m^2 , a throat with cross-sectional area of 0.00031 m^2 is connected to it. If the pressure difference is measured to be 2.4 kPa , what is the speed of the water in the pipe?

Given Data:

$$\text{Cross-sectional area of pipe} = A_1 = 0.0038 \text{ m}^2$$

$$\text{Cross-sectional area of throat} = A_2 = 0.00031 \text{ m}^2$$

$$\text{Pressure difference} = P_1 - P_2 = 2.4 \text{ kPa} = 2.4 \times 10^3 \text{ Pa}$$

$$\text{Density of water} = \rho = 1000 \text{ kg m}^{-3}$$

Required Data:

$$\text{Speed of water in the pipe} = V = ?$$

Solution:

As the speed of water in a pipe in terms of pressure difference is given by:

$$v_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Putting values

$$v_1 = (0.00031) \sqrt{\frac{2(2.4 \times 10^3)}{1000(0.0038)^2 - (0.00031)^2}}$$

$$v_1 = (0.00031) \sqrt{\frac{4.8}{1000(0.000014 - 0.000000096)}}$$

$$v_1 = (0.00031) \sqrt{\frac{4.8}{(1.4 \times 10^{-5} - 0.00961 \times 10^{-5})}}$$

$$v_1 = (0.00031) \sqrt{\frac{4.8}{10^{-5}(1.4 - 0.00961)}}$$

$$v_1 = (0.00031) \sqrt{\frac{4.8 \times 10^5}{1.39}}$$

$$v_1 = (0.00031) \sqrt{34.5 \times 10^4} = 0.18 \text{ m/s}$$

Result:

$$\text{Speed of water in the pipe} = v_1 = 0.18 \text{ m/s}$$

Numerical Problems

Problem 1: Eight equal drops of oil are falling through air with steady velocity of 0.1 ms^{-1} . the drops recombine to form a single drop, what should be the new terminal velocity?

Given Data:

Terminal velocity of single drop $= v_T = 0.1 \text{ m / sec}$

Required Data:

Terminal velocity of single drop formed by the recombination of eight drops $= v'_T = ?$

Solution:

Let "m" represents the mass of single drop and M represents the mass of single drop after the combination of eight drops so we have

$$M = 8m$$

Similarly if "V" represents the volume of single drop, then the volume of the single drop after the combination of eight drops is given by:

$$\Rightarrow \frac{4}{3} \pi R^3 = 8 \frac{4}{3} \pi r^3$$

$$R^3 = 8r^3$$

$$R^3 = (2r)^3$$

$$\sqrt[3]{R^3} = \sqrt[3]{(2r)^3}$$

$$R = 2r$$

Now the terminal velocity of big drop "R" in term of mass "M" is given by:

$$v'_T = \frac{Mg}{6\pi\eta R} \quad \text{Putting values}$$

$$v'_T = \frac{8mg}{6\pi\eta(2r)} = \frac{8}{2} \left(\frac{mg}{6\pi\eta r} \right)$$

$$v'_T = 4 \left(\frac{mg}{6\pi\eta r} \right) \text{----- (i)}$$

As $v_T = \frac{mg}{6\pi\eta r}$ is the terminal velocity of single drop.

So equation (i) becomes

$$v'_T = 4v_T$$

$$v'_T = 4(0.1) = 0.4 \text{ m / sec}$$

New terminal velocity of single drop after recombination $= v'_T = 0.4 \text{ m / sec}$

Problem 2: Water travels through a 9.6 cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5cm. (a) What is the speed of the water coming out of

thenozzle? (b) What diameter nozzle is required to give water speed of 21 m/s?

Given Data:

Diameter of one end of first hose = $d_1 = 9.6\text{cm} = 0.096\text{m}$

Speed of water at end one = $v_1 = 0.3\text{m/sec}$

Diameter of other end of fire hose = $d_2 = 2.5\text{cm} = 0.025\text{m}$

Required Data:

a) Speed of water at other end = $v_2 = ?$

b) For speed $v_2 = 21\text{m/s}$ Diameter of other end $d_2 = ?$

Solution (a):

From equation of continuity we have

$$A_1 v_1 = A_2 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\left(\frac{d_1}{2}\right)^2 v_1 = \left(\frac{d_2}{2}\right)^2 v_2$$

$$\frac{d_1^2 v_1}{4} = \frac{d_2^2 v_2}{4}$$

$$d_1^2 v_1 = d_2^2 v_2 \text{ ----- (i)}$$

$$v_2 = \frac{d_1^2 v_1}{d_2^2}$$

Putting values

$$v_2 = \frac{(0.096)^2 (1.3)}{(0.025)^2} = 19.2 \text{ m/sec}$$

Result:

$$\text{Speed of water at other end} = v_2 = 19.2 \text{ m/sec}$$

Solution (b):

If speed = $v_2 = 21\text{m/sec}$ then find = $d_2 = ?$

Putting values in equation (i)

Equation (i) $d_1^2 v_1 = d_2^2 v_2$

$$(0.096)^2 (1.3) = d_2^2 (21)$$

$$d_2^2 = \frac{(0.096)(1.3)}{21} = 5.7 \times 10^{-4}$$

$$\sqrt{d_2^2} = \sqrt{5.7 \times 10^{-4}}$$

$$d_2 = 2.4 \times 10^{-2} \text{ m} = 2.4 \text{ cm}$$

$$[10^{-2} = \text{centi} = c]$$

Result:

$$d_2 = 2.4 \text{ cm} \quad \text{If} \quad v_2 = 21 \text{ m/sec}$$

Problem 3. A fish tank has dimensions 0.30 m wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 3.0 h, what should the flow speed be in the 3.0 cm diameter input tube for the filter?

Given Data:

Volume of fish tank = $V = \text{width} \times \text{length} \times \text{height}$

$$= (0.30)(1.0)(0.60) = 0.18 \text{ m}^3$$

$$\text{Time taken} = t = 3 \text{ hr} = 3 \times 60 \times 60 = 10800 \text{ sec}$$

$$\text{Diameter} = D = 3.0 \text{ cm} = 0.03 \text{ m}$$

$$\text{Radius} = r = \frac{D}{2} = \frac{0.03}{2} = 0.015 \text{ m}$$

Required Data:

$$\text{Flow speed} = v = ?$$

Solution:

As we know that volume flow rate is given by:

$$\frac{V}{t} = Av$$

$$\because A = \pi r^2$$

$$\frac{V}{t} = \pi r^2 v$$

$$v = \frac{V}{\pi r^2 t}$$

Putting values we get

$$v = \frac{0.18}{(3.14)(0.015)^2 (10800)} = \frac{0.18}{7.6}$$

$$v = 0.024 \text{ m/sec}$$

$$v = 2.4 \times 10^{-2} \text{ m/sec} = 2.4 \text{ cm/sec}$$

Result:

$$\text{Flow speed} = v = 2.4 \text{ cm/sec}$$

Problem 4: A venturi meter is measuring the flow of water; it has a main diameter of 3.5 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm-Hg, what is the speed of the water entering the venturi throat?

Given Data:

$$\text{Main diameter} = D_1 = 3.5 \text{ cm} = 0.035 \text{ m}$$

$$\text{Main radius} = r_1 = \frac{D_1}{2} = \frac{0.035}{2} = 0.018 \text{ m}$$

$$\text{Throat diameter} = D_2 = 1.0 \text{ cm} = 0.01 \text{ m}$$

$$\text{Throat radius} = r_2 = \frac{D_2}{2} = \frac{0.01}{2} = 0.005 \text{ m}$$

$$\text{Area of main section} = A_1 = \pi r_1^2 = (3.14)(0.018)^2 = 0.001 \text{ m}^2$$

$$\text{Area of throat section} = A_2 = \pi r_2^2 = (3.14)(0.005)^2 = 7.85 \times 10^{-5} \text{ m}^2$$

$$\text{Pressure difference} = P_1 - P_2 = 18 \text{ mm of Hg}$$

$$= 18(133) \text{ Pa} = 2400 \text{ Pa}$$

$$\text{Density of water} = \rho = 1000 \text{ kg/m}^3$$

Required Data:

$$\text{Speed of water entering the throat} = V_1 = ?$$

Solution:

We know that the speed of water by venturimeter in terms of pressure is given by:

$$v_1 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{2(2400)}{1000[(0.001)^2 - (7.85 \times 10^{-5})^2]}}$$

$$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4800}{1000[0.1 \times 10^{-5} - 6.2 \times 10^{-10}]}}$$

$$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4800}{1000(9.9 \times 10^{-7})}}$$

$$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4800}{0.00099}}$$

$$v_1 = 7.85 \times 10^{-5} \sqrt{4848485}$$

$$v_1 = 7.85 \times 10^{-5} (2202)$$

$$v_1 = 0.17 \text{ m/sec}$$

Result: Speed of water in pipe = $V_1 = 0.17 \text{ m/sec}$

Problem 5: A small circular hole 6.00 mm in diameter is cut in the side of a large watertank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second.

Given Data:

$$\text{Diameter of hole} = d = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\text{Radius of hole} = r = \frac{d}{2} = \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3} \text{ m}$$

$$\text{Height of tank} = h = 14 \text{ m}$$

Required Data:

a) Speed of efflux = $v = ?$

b) Volume discharged per second = $\frac{\Delta V}{\Delta t} = ?$

Solution (a):

By Torricelli's theorem we have

$$v = \sqrt{2gh} \quad \text{Put values}$$

$$v = \sqrt{2(9.8)(14)}$$

$$v = \sqrt{274.4} = 16.6 \text{ m/sec}$$

Result:

$$\text{Speed of efflux} = v = 16.6 \text{ m/sec}$$

Solution (b):

Now to find $= \frac{\Delta V}{\Delta t} = ?$

As we know that

$$\frac{\Delta V}{\Delta t} = Av \quad (\because A = \pi r^2)$$

$$\frac{\Delta V}{\Delta t} = \pi r^2 v \quad \text{Putting values}$$

$$\frac{\Delta V}{\Delta t} = (3.14)(3 \times 10^{-3})^2 (16.6)$$

$$\frac{\Delta V}{\Delta t} = (3.14)(9 \times 10^{-6})(16.6)$$

Result:

$$\frac{\Delta V}{\Delta t} = 0.000469 \text{ m}^3 / \text{sec}$$

Problem 6: What is the Aerofoil lift (in newtons) due to Bernoulli's principle on a paperplane of wing area 0.01 m^2 if the air passes over the top and bottom surfaces at speeds of 9 m/s and 7 m/s respectively? (Take the density of air as 1.28 kg/m^3 .)

Given Data:

$$\text{Area of wing} = A = 0.01 \text{ m}^2$$

$$\text{Speed at top of wing} = v_2 = 9 \text{ m/sec}$$

$$\text{Speed at below the wing} = v_1 = 7 \text{ m/sec}$$

$$\text{Density of air} = \rho = 1.28 \text{ kg/m}^3$$

Required Data:

$$\text{Aero-foil lift} = F = ?$$

Solution:

We know that $\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

$$P_1 - P_2 = \frac{F}{A} \text{----- (1)}$$

From venturimeter we have

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \text{----- (2)}$$

Comparing equation (1) & (2) we get

$$\frac{F}{A} = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$F = \frac{A}{2} \rho (v_2^2 - v_1^2) \quad \text{Putting values}$$

$$F = \frac{0.01}{2} (1.28)(9^2 - 7^2)$$

$$F = 6.4 \times 10^{-3} (81 - 49) = 6.4 \times 10^{-3} (32)$$

$$F = 204.8 \times 10^{-3} = 0.2048 = 0.2 \text{ N}$$

Result:

$$\text{Aero-foil lift} = F = 0.2 \text{ N}$$

Problem 7: During a windstorm, a 25 m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and air just above the roof, assuming the doors and windows of the house are closed. (The density of air is 1.28 kg/m^3).

Given Data:

Speed of wind inside the home $= v_1 = 0 \text{ m/sec}$

Speed of wind above the roof $= v_2 = 25 \text{ m/sec}$

Density of water $= \rho = 1.28 \text{ kg/m}^3$

Required Data:

Pressure difference $= P_1 - P_2 = ?$

Solution:

As we know that

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad \text{Putting values}$$

$$P_1 - P_2 = \frac{1}{2} (1.28) [(25)^2 - (0)^2]$$

$$P_1 - P_2 = 0.64 [625 - 0]$$

$$P_1 - P_2 = 0.64 \times 625$$

$$P_1 - P_2 = 400 \text{ N/m}^2 = 400 \text{ Pa}$$

Result:

$$\text{Pressure difference} = P_1 - P_2 = 400 \text{ Pa}$$